COLLEGE OF ARTS AND SCIENCES

DEPARTMENT OF MATHEMATICS

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- Engage in community service and conduct researches—especially scientific, technological, business, and social researches—that will benefit the community
- Be a role model in his/her workplace in terms of the WITtian Identity and Core Values

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Modules for Math 0

(Mathematics for Engineering Science and Technology)

Module 1 (Polynomials)

Module Description **:** In this module important type of algebraic expression called a polynomial will be discussed. Polynomials are extremely useful because they are so simple: The variables are only added, subtracted, and multiplied. A variable is a letter used to represent an arbitrary element of a set, while a constant is a letter used to represent a specific element of a set. In this module, variables will represent real numbers almost all the time and complex numbers occasionally. Polynomials will be used repeatedly throughout this module.

Lesson 1 : Fundamental operations on polynomials: Addition, subtraction, multiplication and division of polynomials. Synthetic Division and removing and inserting symbols of grouping.

Introduction: : This lesson will include the discussion on the properties of real numbers, the laws of signs, and perform the four fundamental operations (addition, subtraction, multiplication, and division) applying laws of exponents on multiplication and division and simplifying algebraic expression using the correct way of inserting and removing symbols of grouping

Learning Objectives : To provide the students the ability to perform the four fundamental operations (addition, subtraction, multiplication, and division) of polynomials, applying laws of exponents on multiplication and division, synthetic division, and simplifying algebraic expression using the correct way of inserting and removing symbols of grouping.

Learning Outcomes : Upon completion of this lesson, the student should have:

1. Performed the four fundamental operations: addition, subtraction, multiplication, and division on polynomials.
2. Applied the laws of exponents for multiplication and division of polynomials.
3. Simplified algebraic expressions by removing symbols of grouping.
4. Appreciate the use and application of the four fundamental operations on polynomials.

Time Allotment **: 2 hrs lecture / 3 hrs lab**

Presentation of the Lesson

1.1 Concept of a Polynomial

Polynomial is made up of two terms, namely Poly (meaning "many") and Nominal (meaning "terms."). A polynomial is defined as an expression which is composed of variables, constants and exponents, that are combined using the mathematical operations such as addition, subtraction, multiplication and division (No division operation by a variable). Based on the numbers of terms present in the expression, it is classified as monomial, binomial, and trinomial.

Examples of constants, variables and exponents are as follows:

- **Constants. Example:** **1, 2, 3, etc.**
- **Variables. Example:** **g, h, x, y, etc.**
- **Exponents: Example:** **5 in x^5 etc.**

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1.2 Definition of terms

Variable – is an object or symbol that changes its value in a particular problem or discussion.

Constant – is an object or symbol that does not change its value in a particular problem or discussion.

Algebraic expression – is a constant, a variable or a combination of constants and variables together with the operations of addition, subtraction, multiplication, division, raising to powers or extraction of roots.

Terms – consist of a variable and constant separated by + or – sign.

Polynomial – is an expression which is composed of terms separated by the fundamental operations in Algebra.

Monomial – is an algebraic expression consisting of only one term.

Binomial – is an algebraic expression which consists of two unlike terms.

Trinomial – is an algebraic expression which consists of three unlike terms.

Multinomial – is an algebraic expression which consists of four or more unlike terms.

Literal coefficient – is the symbol or letter in a particular expression.

Numerical coefficient – is a numeral/number in a particular expression.

Degree of a term – is the sum of the exponents of its variables.

Degree of a Polynomial – is the highest exponent found in any non–zero term of the polynomial.

1.3 Kinds of Polynomials

1. **Monomial**

   Example 1 x, 5, $(x - 7)$

2. **Binomial**

   Example 2 $3x^3 - 5x^7$, $2y^2 - 1$, $5a + 2$

3. **Trinomial**

   Example 3 $7x^2 + 5x - 3$, $2x^2y - 5xy^3 - 3xy$, $3a^2b - 5ab^2 + 7ab$

4. **Multinomial**

   Example 4 $x^3 + 3x^2 + 3y + 1$, $y^3 + y^2 + 9y + 27$ $a^3b + a^2b^2 - 7ab^3 + 9$

There are also some expressions, which cannot be considered as polynomials.

Example 5

1. $2x^{-2}$, This is not a polynomial because of the negative exponent –2. An expression is a polynomial if the exponent is a positive integer or zero.
2. $\frac{3}{x-5}$, This is not a polynomial because the term does not follow the form ax^n.

Note: An algebraic expression is a polynomial if and only if all the terms are in the form ax^n; where n is an element of the whole numbers {0, 1, 2, 3, . . .}.

1.4 Degree of Polynomials

In determining the degree of a term of a polynomial, find the sum of the exponents of its variable.

Example 6

$2x^2$ 2nd degree

$3x^2y$ 3rd degree

19 0 degree because $19x^0 = 19$

In determining the degree of a polynomial, get the highest exponent found in any non–zero term of the polynomial

Example 7

$3x^5 - x^2 + 7$ 5th degree

$9m^3 - 4m^2 + 6$ 3rd degree

$3m^4 - 4m^2$ 4th degree

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1.5 Operations on Polynomials

1.5.1 Addition

A. Addition of Monomials

Procedure

1. To add similar or like terms, find the algebraic sum of the numerical coefficients and prefix it to their common literal factor.

   Note: The Addition Laws of Signed Numbers

   a. In adding numbers having the same signs, find their sum and bring down their common sign.

   b. In adding numbers having different signs, find their difference and bring down the sign of the number with the greater numerical value.

2. To add dissimilar or unlike terms, their sum cannot be written as a single term, instead they can only be indicated.

Example 8

Add the following:

a. $3x + 5x = 8x$

b. $(-8y) + (-2y) = -10y$

c. $5ab + (-2ab) = 3ab$

d. $(-8bc) + 8bc = 0$

e. $ab + 3ab = 4ab$

B. Addition of Polynomials

There are two ways of adding polynomials:

1. horizontal form
2. vertical form

In adding polynomials in horizontal form, simply combine similar terms

Example 9

Find the sum of $3x^3 - 5x^2 + 7$ and $-2x^3 + 7x^2 - 5$

Solution

$$\begin{aligned}(3x^3 - 5x^2 + 7) + (-2x^3 + 7x^2 - 5) &= 3x^3 - 5x^2 + 7 - 2x^3 + 7x^2 - 5 \\ &= (3x^3 - 2x^3) + (-5x^2 + 7x^2) + (7 - 5) \\ &= (x^3) + (2x^2) + (2) \\ &= x^3 + 2x^2 + 2\end{aligned}$$

In adding polynomials in vertical form, arrange the terms, so that similar terms or like terms are in the same vertical columns, then add each column.

In some cases, it will be necessary to rewrite the polynomials in ascending or descending powers of one letter.

Using the same example

$3x^3 - 5x^2 + 7$ and $-2x^3 + 7x^2 - 5$

$$\begin{array}{r} 3x^3 - 5x^2 + 7 \\ \underline{-2x^3 + 7x^2 - 5} \\ x^3 + 2x^2 + 2 \end{array}$$

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1.5.2 Subtraction

A. Subtraction of Monomials

Procedure

1. To subtract similar or like terms, change the sign of the subtrahend and proceed as in algebraic addition.
2. The subtraction of dissimilar or unlike terms can only be indicated. However, change the sign of the subtrahend.

Example 10

a. Subtract $-\ 5x$ from $3x$

b. Subtract $2x$ from 0

c. Subtract 0 from $-\ 5ab$

Solution

a. $(3x) - (-5x) = 8x$

b. $(0) - (2x) = -2x$

c. $(-5ab) - (0) = -5ab$

B. Subtraction of Polynomials

Procedure

- To subtract polynomials in horizontal form, change the sign of each term of the second polynomial and proceed to addition.

Example 11

a. Find the difference of $5y^2 - 2y + 3$ and $-\ 8y^2 + 7y - 6$

Solution

$$\begin{aligned}(5y^2 - 2y + 3) - (-\ 8y^2 + 7y - 6) &= (5y^2 - 2y + 3) + (\ 8y^2 - 7y + 6)\\ &= (5y^2 - 2y + 3 + \ 8y^2 - 7y + 6)\\ &= (5y^2 + 8y^2) + (-2y - 7y) + (3 + 6)\\ &= (5 + 8)y^2 + (-2 - 7)y + (9)\\ &= 13y^2 - 9y + 9\end{aligned}$$

b. Subtract $5x^2y + 6xy$ from $-2xy^2 + 4x^2y - 6xy$

Solution

$$\begin{aligned}(-2xy^2 + 4x^2y - 6xy) - (5x^2y + 6xy) &= (-2xy^2 + 4x^2y - 6xy) + (-\ 5x^2y - 6xy)\\ &= (-2xy^2 + 4x^2y - 6xy - 5x^2y - 6xy)\\ &= (-2xy^2) + (4x^2y - 5x^2y) + (-6xy - 6xy)\\ &= -2xy^2 + (4 - 5)x^2y + (-6 - 6)xy\\ &= -2xy^2 - x^2y - 12xy\end{aligned}$$

- To subtract polynomials in vertical form, arrange the terms so that similar or like terms are in the same vertical columns, then change the sign of the subtrahend and proceed as in addition.

In some cases, it will be necessary to rewrite the polynomials in ascending or descending powers of one letter.

Using the same example

c. Find the difference of $5y^2 - 2y + 3$ and $-\ 8y^2 + 7y - 6$

Solution

$$\begin{array}{r} 5y^2 - 2y + 3 \\ (+) \quad (-) \quad (+) \\ -\ 8y^2 + 7y - 6 \\ \hline 13y^2 - 9y + 9 \end{array}$$

d. Subtract $5x^2y + 6xy$ from $-2xy^2 + 4x^2y - 6xy$

Solution

$$\begin{array}{r} -2xy^2 + 4x^2y - 6xy \\ (-) \qquad (-) \\ 5x^2y + 6xy \\ \hline -2xy^2 - x^2y - 12xy \end{array}$$

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1.5.3 Multiplication

Positive Integer Exponents

It can be shown that $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$. These definitions will be consistent with the laws for exponents in this section, which are given here for positive integer exponents.

The Laws for Exponents / Index Laws

1.	$a^n = a.a.a.\dots a$	nth power of a
2.	$a^m a^n = a^{m+n}$	add exponents
3.	$a^n b^n = (ab)^n$	multiply bases
4.	$(a^n)^m = a^{mn}$	multiply exponents
5.	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	divide bases
6.	$\frac{a^m}{a^n} = a^{m-n}$	subtract exponents

Example 12

a. $3^2 3^3 = 3^{2+3} = 3^5 = 243$

b. $\frac{7^5}{7^3} = 7^{5-3} = 7^2 = 49$

c. $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$

d. $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

e. $(3x^2y^2)(4xy^5) = 3 \cdot 4 \cdot x^2 \cdot x \cdot y^2 \cdot y^5 = 12x^3y^7$

f. $\left(\frac{5x^5y^8}{3z^4w^2}\right)^3 = \frac{5^3(x^5)^3(y^8)^3}{3^3(z^4)^3(w^2)^3} = \frac{125x^{15}y^{24}}{27z^{12}w^6}$

A. Multiplication of a Monomial by Another Monomial

To multiply a monomial by another monomial, find the product of their numerical coefficients and add the exponents of the same variable. Index laws are to be applied.

Example 13

a. $(3x^4)(5x^3) = 15x^{4+3} = 15x^7$

b. $(-3x^2)(9x^{-4}) = -27x^{2-4} = -27x^{-2} = \frac{-27}{x^2}$

c. $(3p^2r^3)(-2p^{-2}r) = -6p^{2-2}r^{3+1} = -6p^0r^{3+1} = -6r^4$

d. $(-2s^3t^{-2})^4 = (-2)^4(s^3)^4(t^{-2})^4 = 16s^{12}t^{-8} = \frac{16s^{12}}{t^8}$

B. Multiplication of a Polynomial by a Monomial

Procedure

1. To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial using the distributive law of multiplication.
2. If possible, arrange the answer in ascending or descending powers of one of the letters.

Example 14

a. $2x^2y(3x^2y - 2xy^3 + 5y + 3) = 6x^4y^2 - 4x^3y^4 + 10x^2y^2 + 6x^2y$

b. $4x^3y^2z^4(-5xy^2z^2 + 3x^3z - y^4z^5) = -20x^4y^4z^6 + 12x^6y^2z^5 - 4x^3y^6z^9$

c. $-3a^3b^2c^4(2abc - 5a^{-2}b^3c^{-4} - 2a^3) = -6a^4b^3c^5 + 15ab^5c^0 + 6a^6b^2c^4$

$= -6a^4b^3c^5 + 15ab^5 + 6a^6b^2c^4$

C. Multiplication of a Polynomial by Another Polynomial

Two methods may be used in this process: Distributive law of multiplication and the long method. In multiplying polynomials, always arrange the given expressions in either descending or ascending order.

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- **Using the distributive law of multiplication**

  Distribute or multiply each term of the multiplier by the given multiplicand

d. Multiply $(2x^2 + 2x + 1)$ by $(2x + 1)$

$$= 2x(2x^2 + 2x + 1) + 1(2x^2 + 2x + 1)$$
$$= (2x \cdot 2x^2) + (2x \cdot 2x) + (2x \cdot 1) + (1 \cdot 2x^2) + (1 \cdot 2x) + (1 \cdot 1)$$
$$= 4x^3 + (4x^2 + 2x^2) + (2x + 2x) + 1$$
$$= 4x^3 + 6x^2 + 4x + 1$$

e. Multiply $(4a^2 + 2a - 2)$ by $(2a^2 - a + 1)$

$$= 2a^2(4a^2 + 2a - 2) + -a(4a^2 + 2q - 2) + 1(4a^2 + 2q - 2)$$
$$= (2a^2 \cdot 4a^2) + (2a^2 \cdot 2a) + (2a^2 \cdot -2) + (-a \cdot 4a^2) + (-a \cdot 2a) + (-a \cdot -2) + (1 \cdot 4a^2) + (1 \cdot 2a) + (1 \cdot -2)$$
$$= 8a^4 + 4a^3 - 4a^2 - 4a^3 - 2a^2 + 2a + 4a^2 + 2a - 2$$
$$= 8a^4 - 2a^2 + 4a - 2$$

- **Using the Long Method**

  The process of multiplying polynomials using the long method, is similar to the process used in arithmetic; the only difference is, we start at the left side of the expression, going to the right side.

  Using the same examples

f. Multiply $(2x^2 + 2x + 1)$ by $(2x + 1)$

$$\begin{array}{l} 2x^2 + 2x + 1 \\ \underline{\qquad\quad 2x + 1} \\ 4x^3 + 4x^2 + 2x \\ \underline{\qquad\quad 2x^2 + 2x + 1} \\ 4x^3 + 6x^2 + 4x + 1 \end{array}$$

g. Multiply $(4a^2 + 2a - 2)$ by $(2a^2 - a + 1)$

$$\begin{array}{l} 4a^2 + 2a - 2 \\ \underline{2a^2 - a + 1} \\ 8a^4 + 4a^3 - 4a^2 \\ \qquad -4a^3 - 2a^2 + 2a \\ \underline{\qquad\qquad\quad 4a^2 + 2a - 2} \\ 8a^4 \qquad\quad - 2a^2 + 4a - 2 \end{array}$$

1.5.4 Division

- **Division of a Polynomial by a Monomial**

  In dividing a polynomial by a monomial, find the quotient of their numerical coefficients and apply the exponential law of division.

Example 15

a. Divide $(25x^3y^2 + 15x^2y^2 + 5xy)$ by $5xy$

$$= \frac{25x^3y^2}{5xy} + \frac{15x^2y^2}{5xy} + \frac{5xy}{5xy}$$
$$= 5x^2y + 3xy + 1$$

b. Divide $(42a^5b^3 + 36a^3b + 12ab^3)$ by $3ab$

$$= \frac{42a^5b^3}{3ab} + \frac{36a^3b}{3ab} + \frac{12ab^3}{3ab}$$
$$= 14a^4b^2 + 12a^2 + 4b^2$$

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- **Division of a Polynomial by Another Polynomial**

In dividing a polynomial by another, follow the following procedures:

1. To divide polynomials, arrange the terms of both the dividend and divisor in either ascending or descending powers of some common letter.
2. Divide the first term of the dividend by the first term of the divisor and write the result as the first term of the quotient.
3. Multiply the first term of the quotient by the divisor and write this product under the dividend, keeping similar terms under each other. Then subtract the product from the dividend. The difference will be the new dividend.
4. Repeat steps 2 and 3 until you obtain the final answer or the quotient. (Note: The answer is final when the exponent of the dividend is less than the exponent of the divisor).
5. Check your answer by multiplying the quotient by the divisor plus the remainder.

$$Dividend = Quotient \cdot Divisor + \mathrm{Re}\,mainder$$

Example 16

a. Divide $(x^2 - 2x + 1)$ by $(x - 1)$

$$\begin{array}{r|l} & x-1 \\ x-1 & x^2-2x+1 \\ & \underline{x^2-x} \\ & \quad -x+1 \\ & \quad \underline{-x+1} \\ & \qquad 0 \end{array}$$

b. Divide $(4x^3 - 3x^2 + 2x + 1)$ by $((x^2 - x + 1)$

$$\begin{array}{r|l} & \qquad 4x+1 \\ x^2-x+1 & 4x^3-3x^2+2x+1 \\ & \underline{4x^3-4x^2+4x} \\ & \qquad x^2-2x+1 \\ & \qquad \underline{x^2-\ \ x+1} \\ & \qquad\quad -x \end{array}$$

Since the exponent of the new quotient is less than the exponent of the first term of the divisor, we stop. The expression $-x$ shall be treated as the remainder. Therefore,

$$\frac{4x^3-3x^2+2x+1}{x^2-x+1} = 4x+1-\frac{x}{x^2-x+1}$$

1.5.5 Synthetic Division:

To divide a polynomial by a binomial of the type $x - r$, write the numerical coefficients of the terms of the given polynomial arranged according to descending powers of x, be sure to supply each missing power with zero as its coefficient.

In the first line, write the numerical coefficients of the terms thus $a_0, a_1, a_2, \cdots a_n$. Write a_0 in the first place in the third line. Multiply this sum by r, replace the product in the second line under a_2 and add. Continue until the last sum in the third line is obtained. This is the remainder and the preceding numbers from left to right are the coefficients of the powers of x in the quotient in descending order.

Example 17

a. Divide $x^3 - 5x^2 + 10x - 8$ by $x - 2$

Solution

1	– 5	10	– 8	
	2	– 6	8	2
1	– 3	4	0	

Answer: $x^2 - 3x + 4$

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1.5.6 Symbols of Grouping (Signs of Aggregations)

Grouping symbols are often necessary to express clearly the meaning of certain expressions and also to indicate the order in which the operations are to be performed.

The common Symbols of Grouping are:

1. Parentheses ( )
2. Brackets [ ]
3. Braces { }
4. Bar, Vinculum ——

Rules in Removing or Inserting Symbols of Grouping

1. If a factor is outside the sign of grouping, expand by multiplying the factor by each of the terms inside the grouping symbol.

   Example 17

   $$2xy(x^2 - 4xy^3 + 3x^2y) = 2x^3y - 8x^2y^4 + 6x^3y^2$$

2. In removing or inserting symbols of grouping preceded by a plus (+) sign, remove or insert the symbol and maintain the terms inside as is.

   Example 18

   $$3x + (2x - 3y) = 3x + 2x - 3y = 5x - 3y$$

   $$+a - b = +(a - b)$$

3. In removing or inserting symbols of grouping preceded by a minus (–) sign, remove or insert the symbol and change all the signs of the terms inside from $(+)$ to $(-)$ or from $(-)$ to $(+)$,

   Example 19

   $$3x - (2x - 3y) = 3x - 2x + 3y$$
   $$= x + 3y$$
   $$+a - b = -(-a + b)$$

If there are more than one set of grouping symbols in an expression, to remove the symbols, start with the innermost symbol applying the first three rules $(1), (2)\ and\ (3)$.

Example 20

$$x\{x - 3(y - 2z) + 6\} = x\{x - 3y + 6z + 6\}$$
$$= x^2 - 3xy + 6xz + 6x$$

Exercise 1

I. Identify the numerical coefficient and the literal coefficient in each of the following expressions. Give the degree of each:

1. $-7x$
2. $x^2y^3 - 3x$
3. $3m^2n^2 + 8$
4. $\frac{3}{4}(a + b)$
5. $3(x + y) - 6x^3y^5 - 5x^2y^2 + 7$
6. $\frac{1}{2}(x^3 + x + 2x^2) + t^4$
7. $x - y + x^2 - y^5$
8. $z^4 + z^3 - 2z^2 + z - 9$
9. $3x^5y - 5x^2yz^2 + 2s^2t^4u^3$
10. $x^3 - y^3 + x^2 - y^2$

II. Combine the following expressions:

1. $8x - 12y - 7x - 8y + 6 - 7x - 2x + 5y$
2. $13xy + x - 9x + 4y - 6x - 9xy - 14x - 5xy$
3. $4x^2y - 5x^2 - 5xy - 11x^2y + 7x^2 + 8xy - 9x^2y$
4. $5ab + 12abc - ab - 12a^2b + 4abc - 3ab + 16a^2b$
5. $3.5s^3t^2 + 4.8st - 5.3s^3t^2 - 15.5st - 6.9s^3t^2$

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III. Add the following expressions:

1. $2x - 2y - 5z$
   $\underline{2x + 4y + 3z}$

2. $3a + 4b - 6c$
   $\underline{5a + 3b - 3r}$

3. $2a - 3p + 7x$
   $3a + 5p - 8x$
   $\underline{-4a + 4p + 3x}$

4. $8pq + 4pr - 2qr$
   $8pq - pr + 7qr$
   $\underline{-3pq + 2pr - qr}$

5. $2x + 3y - 4z$, $-3x - 8y - 3z$; $5x - 4y + 6z$
6. $3p^2q + 4pq + 5pq^2$ $-7p^2q - 6pq + 3pq^2$ $5p^2q - 7pq^2 + 4pq$
7. $2a^2 + 4ab + 3ab^2$ $-3ab^2 - 3a^2 - 4ab$ $4a^2 - 6ab^2$
8. $7r^2s + 2r^2s^2 + 3s^2$ $-12r^2s - 5r^2s^2 - 2s^2$ $8r^2s + 2s^2 + 7r^2s^2$
9. $2x^2 + 4xy - 3y^2$ $2x^2 + xy - 3y^2$ $y^2 - xy - 4x^2$
10. $3x^2 + 5y^2 - 3x - y + 2$ $y^2 - 2x^2 - x - 3y - 5$ $x^2 + y^2 + 4y$

IV. Subtract the second expression from the first:

1. $5a^2 + 9ab$ $2a^2 - 8b$
2. $14r + 15s - 5q$ $13r + 12s + 11q$
3. $31s^2 + 19y^2 - 18z^2$ $16x^2 - 18z^2$
4. $4y^2 - 5x^2 + 4z^2$ $-5z^2 + 8y^2$
5. $4.8m - 5.3n + 4.1p$ $8.9m^2 - 2.4n + 5.7p$

V. Subtract $16x + 25y$ from the sum of $-5x + 13y + 19z$ and $3x + 2y$

VI. Subtract the sum of $-11p + 8q + 5z$ and $-3p + 2q - 6z$ from $4p + 2q - z$

VII. Remove grouping symbols and then combine like terms:

1. $12a - [6a - (12a - b)] + 3b$
2. $10a + (6b - 7c) - [2a - 3b - (4a - 4b + 3c) + 9c] - 18$
3. $8\{-5x + 3[2x - 3(x - 9) + 3xy] - 3y - (8y + 2z)\}$
4. $8xy - 5(3ax - ay - \{32xy + 2ay\})$
5. $9x^2 - 8\{x^2 + 2y^2 - 9(x^2 - 1) + 9(x^2 - 3y^2)\}$
6. $5a\{a^2 - a[2 - 5(A + 2) + 7] - 3a^2\}$
7. $7a^3 - 5\{[5a^2 + 2(a - 1) - 14] - a^2 + 5(2a - a^2)\}$
8. $3a - \{2b - 4[c + 2b - 3(c - 5a) + 6b] + 4a\}$
9. $21x^2 - 2(3x^2 - 3x + 7\{3x^2 - 2x - 1\} + \{3x^2 - 6x\})$
10. $a^2b - 2\{a^2b - 3b^2 - 2(a^2b + 3b^2 + 2b^2 - a^2b) - 3a^2b\}$

VIII. Applying the laws of exponents, simplify each of the following:

1. $x^5 \cdot x^7$
2. $-s^{11} \cdot s^5$
3. $(z^2)(z^3)(2z^4)$
4. $(x^2y^4)(x^4y^7)$
5. $(-5b^2c^5)(6b^4c^{-2})(b^3c^4)$
6. $(x + y)(x + y)$
7. $(v^5w^4x^3)(v^2w^3x^4)$
8. $2^2 \cdot 2^4 \cdot 2$
9. $(p^2 + 1)(p^2 + 1)$
10. $(-2.5c^3d^4e^2)(-0.7c^4d^3e^6)$

IX. Determine the product of the following:

1. $(4a^2)(8a^3)$
2. $(125x^6)(3x^3y^3)$
3. $\left(\frac{1}{2}x^3\right)(4x^4 - 8x^3 + 6x)$
4. $(-8x^3y^6)(5x^3y^2)$
5. $(-2x^2y^3)(-3xy^2 - 4x^3y^2)$
6. $2x^2y^2(3y + 2x) - 3x^2(4x - y)$
7. $(4x - 3y)(2x + 3y)$
8. $(2x^2 - 5)(2x + 3)$
9. $(3x - 5)(2x + 3)$
10. $(3x^2 - 3)(3x + 3)$

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11. $(4x^2 - 2x + 1)(-x^2 - 2x + 3)$
12. $(x^3 + 4x - 5)(x^2 - 2x - 4)$
13. $(5x^3 + 2x - 4)(4x^2 - 5x - 6)$
14. $(-4x^3 + 4x^2)(2x^2 - 6x - 7)$
15. $(2x^3 + 4x - 5)(x^3 - 2x^2 - 4x + 1)$
16. $(x + y - 3)(x - 3y + 4)$
17. $(2a + b)(3a^2 - 4ab - 3b^2)$
18. $(2x - 3y + 5)(3x + 6y - 4)$
19. $(x^2 + y^2 - 2)(2x^2 - 2y^2 - 1)$
20. $(x - y - z)(x - y + z)$

X. Determine the quotient of the following:

1. $\frac{-50x^9}{25x^6}$
2. $\frac{81a^{12}b^{12}c^{24}}{27a^{10}b^{10}c^{24}}$
3. $\frac{76x^6y^6}{4x^3y^5}$
4. $\frac{26x^5y^6z^3}{2x^2y^6z^2}$
5. $\frac{75x^3y^5}{25x^2y^4}$
6. $\frac{8x^3-27}{2x-3}$
7. $\frac{2a^6+15a^5-18a^2}{3a^2}$
8. $\frac{27x^7y^6-6x^6y^7-21x^5y^6}{3x^3y^8}$
9. $\frac{4a^2b^2-8a^3b^3+2a^4b^4}{2a^2b^2}$
10. $\frac{18x^5y^4-30x^4y^3-36xy^2}{3x^3y^2}$
11. $\frac{8x^3-2x^2-x-1}{x+2}$
12. $\frac{(3x+5y)^0}{(x+y)^0}$
13. $\frac{3x^2-11x+8}{x-2}$
14. $\frac{6x^3-11x^2+6x+2}{2x^2-3x+1}$
15. $\frac{(3x^4-10x^2-5x+2)(x^5+32)}{(x+2)}$
16. $\frac{x^3+3x^2+3x+1}{x+1}$
17. $\frac{3x^3-4x^2-5x+3}{x^2+x-1}$
18. $\frac{18x^4-12x^3+2x^2-5x+3}{2x-3}$
19. $\frac{3x^6-11x^4+7x^2-3}{x^2-3}$
20. $\frac{12x^5-6x^4+6x^3+x^2+2}{12x^2+13}$

XI. Applications:

1. If the length of a rectangle is $(x^2 + 3x - 5)$ units and its width is $(3x - 1)$, what expression represents its area?
2. If $(3x - 1)$ represents the length of a side of a square, what expression represents its area? What represents its perimeter?
3. If $(2x + 1)$ donuts are sold for $(6x + 5)$ pesos each, what is the cost of the desks?
4. If an apple costs 5x – 3 pesos, how much would a dozen cost?
5. If a jeep can carry 18 passengers, how many can x + 5 jeeps carry?

XII. Key Concept

Be certain that you understand and can use each of the following words, ideas, and indicated procedures:

Algebraic expression	Degree of term	Polynomial
Bar, vinculum	Dissimilar terms	Similar terms
Binomial	Literal coefficient	Symbols of grouping
Braces	Monomial	Terms
Brackets	Multinomial	Trinomial
Constant	Numerical coefficient	Variable
Degree of polynomial	Parentheses	

References and Learning Materials

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Course Outline	No. of Hours	
Part I: Algebra		
1. Polynomials: Four Fundamental Operations	8 hours	
2. Special Products	3 hours	
3. Factoring	6 hours	
4. Fractions	6 hours	
5. Exponents and Radicals	4 hours	
6. Linear Equations	4 hours	
7. Systems of Linear Equations	3 hours	
8. Quadratic Equations	3 hours	
9. Logarithmic and Exponential Functions	3 hours	
Part II: Trigonometry		
1. Circular & Trigonometric Functions	3 hours	(Midterm Examination)
2. Solution of Right Triangles & Applications	3 hours	
3. Solution of Oblique Triangles	3 hours	
4. Trigonometric Identities	3 hours	
Part III: Analytic Geometry		
1. The Rectangular Coordinate System	7 hours	
2. The Straight Line	6 hours	
3. The Circle and Parabola	5 hours	
Part IV: Solid Mensuration		
1. Plane Figures	3 hours	
2. Solids for Which Volume = Bh	3 hours	(Pre – Final Examination)
3. Solids for Which Volume = 1/3 Bh	4 hours	
4. The Sphere and Hemisphere	4 hours0	(Final Examination)

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