

Finite Math

GRADE 11 REVIEWER - 1ST SEMESTER - FINALS

MATRIX (A_{ij})

- A rectangular array or table of numbers, symbols, or expressions arranged in rows and columns.
- Developed by **Arthur Cayley**, a British-American mathematician who published *Memoirs on the Theory of Matrices* in 1858.
- The word **Matrix** itself came from Arthur Cayley and Joseph Sylvester in the 19th century.
 - Sylvester used it to describe an array of numbers that gave rise to different determinants.
- **Rows**
 - Horizontal lines
 - Denote as i
- **Columns**
 - Vertical lines
 - Denote as j
- **Entries / Elements**
 - Each individual values in a matrix
- **Dimensions of Matrices / Order of Matrices**
 - Refers to the size of the rectangular array, defined by its number of rows and columns.

TYPES OF MATRICES

- **Row**
 - A matrix that has only **one row**.
- **Column**
 - A matrix that has only **one column**.
- **Zero / Null**
 - All the elements or entries are **zero**.
- **Square**
 - The number of rows and columns are **equal**
- **Rectangular**
 - The number of rows is **not equal** to the number of columns.
- **Diagonal**
 - A square matrix in which all the elements outside the main diagonal are zero.
- **Identity**
 - A square matrix with ones on the main diagonal and zeros everywhere else.
- **Upper Triangular**
 - A square matrix in which all the elements **below** the main diagonal are zero.
- **Lower Triangular**
 - A square matrix in which all the elements **above** the main diagonal are zero.
- **Symmetric**
 - Square matrix that is its own mirror across the first row and column.
 - $A^T = A$
- **Skew-Symmetric**
 - Square matrix where elements on opposite sides of the diagonal are **additive inverses** (negatives) of each other.
 - $A^T = -A$

ADDITION AND SUBTRACTION OF MATRICES

- **Addition**
 - Add elements of matrices in the same direction/location.
 - **Ex.**

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 2 & 6 \end{bmatrix} \\ = \begin{bmatrix} 2+5 & 4+0 \\ 1+2 & 3+6 \end{bmatrix} \\ = \begin{bmatrix} 7 & 4 \\ 3 & 9 \end{bmatrix}$$

- **Subtraction**
 - Subtract elements of matrices in the same direction/location.
 - **Ex.**

$$\begin{bmatrix} 8 & 10 \\ 6 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \\ = \begin{bmatrix} 8-3 & 10-2 \\ 6-4 & 5-1 \end{bmatrix} \\ = \begin{bmatrix} 5 & 8 \\ 2 & 4 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

- **Scalar multiplication**
 - Multiplying an entire matrix by a single real number.
 - **Ex.**

$$3 \cdot \begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 12 & 15 \end{bmatrix}$$

- **Matrix multiplication**
 - Multiply elements of a row from the first matrix by the corresponding elements of a column from the second matrix, then add the products together.
 - **Ex.**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$C_{11} (1 \cdot 5) + (2 \cdot 7) = 5 + 14 = 19$$

$$C_{12} (1 \cdot 6) + (2 \cdot 8) = 6 + 16 = 22$$

$$C_{21} (3 \cdot 5) + (4 \cdot 7) = 15 + 28 = 43$$

$$C_{22} (3 \cdot 6) + (4 \cdot 8) = 18 + 32 = 50$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

MATRIX TRANSPOSITION (A^T)

- The transpose of a matrix is a new matrix created by swapping the rows and columns of the original matrix.
- **Ex.**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

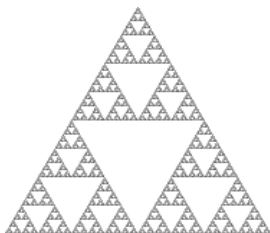
- **Properties of Matrix Transposition**
 - **Addition**
 - $(A + B)^T = A^T + B^T$
 - **Subtraction**
 - $(A - B)^T = A^T - B^T$
 - **Scalar Multiplication**
 - $(kA)^T = kA^T$
 - **Multiplication**
 - $(AB)^T = B^T A^T$
 - **Transposed Matrix**
 - $(A^T)^T = A$

PROPERTIES OF MATRIX OPERATIONS

- **Commutative Property**
 - The order of operands doesn't change the result of the operation.
 - **Matrix multiplication is NOT commutative.**
 - **Addition IS commutative.**
 - $A + B = B + A$
 - **Associative Property**
 - Property that allows us to regroup operands without changing results.
 - $(A + B) + C = A + (B + C)$
 - Applicable for **Scalar Multiplication**
 - $r(sA) = (rs)A$
 - $r(AB) = (rA)B = A(rB)$
 - Applicable for **Matrix Multiplication**
- NOTE/S:** Scalar number should always go left of the matrix to perform scalar multiplication.
- **Distributive Property**
 - Describes how multiplication plays with addition.
 - Applicable for matrix multiplication
 - **Additive Identity**
 - Zero/Null matrix is the additive identity in matrix addition.
 - A matrix multiplied by a zero matrix is equal to a zero matrix.
 - **Additive Inverse**
 - Adding a matrix to its negative results in a Zero Matrix.
 - **Multiplicative Identity**
 - Identity matrix is a square matrix with ones on main diagonal and zeros elsewhere.
 - Also serves as the multiplicative identity.

FRACTALS

- A complex geometric shape or mathematical pattern that exhibits infinite detail, self-similarity, and recursive scaling.
- Fractals cannot be measured easily using standard dimensions.
- Typically created through **iteration**—applying a simple mathematical rule or geometric operation over and over again.
- Fractals have **non-integer (fractional) dimensions**, reflecting how they fill space more than a line but less than a solid surface.
- **Ex.**



DETERMINANT OF MATRICES $|A|$

- Denoted as $|A|$ or $\det(A)$
- A special scalar value that can be computed exclusively from is a **square matrix**.
- **For a 1x1 matrix**, its element is its determinant.
- **For a 2x2 matrix**
 - $ad - bc = |A|$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

PROPERTIES OF DETERMINANTS

1. The determinant of a matrix is equal to the determinant of a transpose.
2. The determinant of an identity matrix is always equal to 1.

$$\begin{aligned} |A| &= \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\ &= (1)(1) - (0)(0) \\ &= 1 \end{aligned}$$

3. If a matrix has a row or column with **all zeros**, its determinant is 0.
4. If 2 rows or columns are **identical**, its determinant is 0.

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ &= (1)(2) - (1)(2) \\ &= 0 \end{aligned}$$

5. Interchanging any 2 rows or columns of a matrix changes the sign of its determinant.

LAPLACE EXPANSION METHOD

- Can be used in getting the determinant of larger matrices.
- In this case, we are commonly asked to solve for 3x3 matrices.
- **Steps in solving**
 1. Get the **first element**, then remove the row and column that involves the first entry.

$$A = \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$

2. Rearrange.

$$|A| = A_1 \cdot \begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix}$$

3. Repeat steps for first elements of each column.
 - Follow a pattern of subtracting, **then** adding.

$$|A| = A_1 \cdot \begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} - B_1 \cdot \begin{vmatrix} A_2 & C_2 \\ A_3 & C_3 \end{vmatrix} + C_1 \cdot \begin{vmatrix} A_2 & B_2 \\ A_3 & B_3 \end{vmatrix}$$

RULE OF SARRUS METHOD

- Also referred to as basketweave method because of the way it's used to solve.
- Can only be used in 3x3 matrices.
- **Steps in solving**
 1. Copy the first two columns and paste them as a 4th and 5th column on the right side

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}$$

2. Get the 1st diagonals

$$\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}$$

3. Get the 2nd diagonals

$$\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}$$

4. Rearrange and subtract

$$\begin{array}{r} \left[\begin{array}{cccccc} a_{11} & a_{22} & a_{33} & + & a_{12} & a_{23} & a_{31} & + & a_{13} & a_{21} & a_{32} \end{array} \right] \\ - \\ \left[\begin{array}{cccccc} a_{12} & a_{21} & a_{33} & + & a_{11} & a_{23} & a_{33} & + & a_{12} & a_{22} & a_{32} \end{array} \right] \end{array}$$

GOD BLESS!

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"Prayers mo lang sapat na"

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