

COMPLETE REVIEWER

Exponents, Imaginary Numbers, and Radicals

This reviewer covers the topics from the provided notes: Laws of Exponents, Index Laws, Imaginary Numbers, and Radicals, with formulas, rules, examples, and a practice test.

I. LAWS OF EXPONENTS

An exponent tells how many times a base is multiplied by itself.

General form: $a^n = a \times a \times \dots \times a$

1. Product Rule / Multiplication

Formula: $a^m \cdot a^n = a^{m+n}$

Rule: Same base $\rightarrow$ keep the base and ADD the exponents.

Example: $x^3 \cdot x^5 = x^8$

2. Power of a Power

Formula: $(a^m)^n = a^{mn}$

Rule: Multiply the exponents.

Example: $(x^3)^4 = x^{12}$

3. Quotient Rule / Division

Formula: $a^m / a^n = a^{m-n}$, $a \neq 0$

Rule: Same base $\rightarrow$ keep the base and SUBTRACT the exponents.

Example: $x^8 / x^3 = x^5$

4. Power of a Product

Formula: $(ab)^n = a^n b^n$

Rule: Distribute the exponent to every factor.

Example: $(2x)^3 = 8x^3$

5. Power of a Quotient

Formula: $(a/b)^n = a^n/b^n$

Rule: Apply the exponent to both numerator and denominator.

Example: $(2x/3)^2 = 4x^2/9$

6. Zero Exponent

Formula: $a^0 = 1$, $a \neq 0$

Example: $7^0 = 1$

7. Negative Exponents

Formula: $a^{-n} = 1/a^n$

Rule: A negative exponent means take the reciprocal.

Example: $x^{-3} = 1/x^3$

8. Fractional Exponents

Formula: $a^{m/n} = \sqrt[n]{a^m}$

Examples: $x^{1/2} = \sqrt{x}$; $x^{1/3} = \sqrt[3]{x}$; $27^{2/3} = 9$

Quick Index Laws Table

Law	Formula	Action
Product	$a^m \cdot a^n = a^{(m+n)}$	Add
Quotient	$a^m / a^n = a^{(m-n)}$	Subtract
Power of Power	$(a^m)^n = a^{(mn)}$	Multiply
Power of Product	$(ab)^n = a^n b^n$	Distribute
Power of Quotient	$(a/b)^n = a^n/b^n$	Distribute
Zero Power	$a^0 = 1$	Becomes 1
Negative Power	$a^{-n} = 1/a^n$	Reciprocal
Fractional Power	$a^{(m/n)} = \sqrt[n]{a^m}$	Convert to radical

II. IMAGINARY NUMBERS

Definition: $i = \sqrt{-1}$, therefore $i^2 = -1$.

Powers of i repeat every four powers:

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

Remainder guide: exponent $\div 4 \rightarrow$ remainder 0: 1; 1: i ; 2: -1 ; 3: $-i$.

Example: $i^{23} = i^3 = -i$.

Negative square roots: $\sqrt{-a} = i\sqrt{a}$.

Example: $\sqrt{-72} = 6i\sqrt{2}$.

Complex number: $a + bi$, where a is the real part and bi is the imaginary part.

Add/Subtract: combine like terms. Example: $(3+4i)+(5+2i)=8+6i$.

Multiply: use distributive property/FOIL and replace i^2 with -1 . Example: $(2+i)(3+4i)=2+11i$.

III. RADICALS

A radical is an expression containing a root symbol.

In $\sqrt[3]{8x^2}$: 3 = index, radical sign = root symbol, $8x^2$ = radicand.

General root relationship: $\sqrt[n]{a} = b$ if and only if $b^n = a$.

1. Product of Radicals

Formula/Rule: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

Example: $\sqrt{3} \cdot \sqrt{12} = \sqrt{36} = 6$.

2. Quotient of Radicals

Formula/Rule: $\sqrt[n]{a} / \sqrt[n]{b} = \sqrt[n]{a/b}$

Example: $\sqrt{18} / \sqrt{2} = \sqrt{9} = 3$.

3. Power of a Radical

Formula/Rule: $(\sqrt[n]{a})^m = a^{m/n}$

Example: $(\sqrt{3})^2 = 3$.

4. Radical of a Power

Formula/Rule: $\sqrt[n]{a^n} = a$; for square roots, $\sqrt{a^2} = |a|$.

Example: $\sqrt{x^2} = |x|$.

5. Simplifying Radicals

Formula/Rule: Look for perfect powers inside the radicand.

Example: $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$.

6. Adding/Subtracting Radicals

Formula/Rule: Only like radicals can be combined.

Example: $3\sqrt{5} + 7\sqrt{5} = 10\sqrt{5}$. But $\sqrt{2} + \sqrt{3}$ cannot be combined.

IV. RATIONALIZING RADICALS

If a radical is in the denominator, multiply by a suitable radical so the denominator becomes rational.

Example: $1/\sqrt{2} \times \sqrt{2}/\sqrt{2} = \sqrt{2}/2$.

V. EXPONENTS $\leftrightarrow$ RADICALS

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = \sqrt[n]{a^m}$$

Examples: $x^{1/2} = \sqrt{x}$; $x^{3/4} = \sqrt[4]{x^3}$; $\sqrt[5]{x^2} = x^{2/5}$.

VI. PRACTICE TEST

A. Laws of Exponents

1. $x^4 \cdot x^7$

2. y^9 / y^3

3. $(a^3)^4$

4. $(2x)^3$

5. $(x/3)^2$

6. z^0

7. x^{-4}

8. $16^{3/4}$

Answer Key: 1. x^{11} | 2. y^6 | 3. a^{12} | 4. $8x^3$ | 5. $x^2/9$ | 6. 1 | 7. $1/x^4$ | 8. 8

B. Imaginary Numbers

1. i^8

2. i^{15}

3. i^{24}

4. $\sqrt{-49}$

5. $\sqrt{-18}$

6. $(3+2i)+(4+5i)$

7. $(7+3i)-(2+i)$

8. $(2+i)(3+4i)$

Answer Key: 1. 1 | 2. $-i$ | 3. 1 | 4. $7i$ | 5. $3i\sqrt{2}$ | 6. $7+7i$ | 7. $5+2i$ | 8. $2+11i$

C. Radicals

1. $\sqrt{36}$

2. $\sqrt{72}$

3. $\sqrt{200}$

4. $\sqrt[3]{54}$

5. $\sqrt{3} \cdot \sqrt{12}$

6. $\sqrt{18}/\sqrt{2}$

7. $3\sqrt{5}+7\sqrt{5}$

8. $\sqrt{12}+\sqrt{27}$

Answer Key: 1. 6 | 2. $6\sqrt{2}$ | 3. $10\sqrt{2}$ | 4. $3\sqrt[3]{2}$ | 5. 6 | 6. 3 | 7. $10\sqrt{5}$ | 8. $5\sqrt{3}$

VII. EXAM CHEAT SHEET

When you see	Remember	Example
Multiply same bases	ADD exponents	$x^2 \cdot x^5 = x^7$
Divide same bases	SUBTRACT exponents	$x^7 / x^2 = x^5$
Power raised to power	MULTIPLY exponents	$(x^2)^5 = x^{10}$
Power outside parentheses	DISTRIBUTE	$(xy)^3 = x^3y^3$

When you see	Remember	Example
Negative exponent	RECIPROCAL	$x^{-2} = 1/x^2$
Zero exponent	1	$x^0 = 1$
Fraction exponent	RADICAL	$x^{(1/2)} = \sqrt{x}$
Negative under square root	USE i	$\sqrt{(-25)} = 5i$
Simplifying radicals	Look for perfect powers	$\sqrt{72} = 6\sqrt{2}$
Adding radicals	Must be like radicals	$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$