

FELICIANO & UY CALCULUS

Master Formula Reviewer + Worked Examples

Study guide note: This PDF is an original study guide summarizing standard calculus formulas and methods associated with the Feliciano & Uy calculus material. It is not a reproduction of the textbook.

I. Algebra & Functions

Laws of Exponents

Formula / Rule	Formula	Used for	Sample
Product	$a^m a^n = a^{(m+n)}$	Use when multiplying powers with the same base.	$x^3 x^4 = x^7$
Quotient	$a^m/a^n = a^{(m-n)}$	Use when dividing powers with the same base.	$x^7/x^3 = x^4$
Power of a Power	$(a^m)^n = a^{(mn)}$	Use for nested powers.	$(x^3)^4 = x^{12}$
Negative Exponent	$a^{-n}=1/a^n$	Rewrite negative powers as reciprocals.	$x^{-3}=1/x^3$
Fractional Exponent	$a^{(m/n)}=\text{root}_n(a^m)$	Convert between radicals and powers.	$x^{(3/2)}=x \sqrt{x}$

Functions

Formula / Rule	Formula	Used for	Sample
Function evaluation	$y=f(x)$	Substitute a given x-value into a function.	$f(x)=x^2-3x+2$; $f(4)=6$
Composition	$(f \circ g)(x)=f(g(x))$	Substitute one function into another.	$f=x^2+1$, $g=2x \rightarrow f(g)=4x^2+1$

II. Limits & Continuity

Limit laws

Formula / Rule	Formula	Used for	Sample
Direct substitution	$\lim_{x \rightarrow a} f(x)=f(a)$ (when continuous)	Use first for ordinary polynomial/rational limits.	$\lim_{x \rightarrow 2} (x^2+3x+1)=11$
Sum/Difference	$\lim(f \pm g)=\lim f \pm \lim g$	Split sums/differences.	$\lim_{x \rightarrow 1} (x^2+x)=2$
Product	$\lim(fg)=(\lim f)(\lim g)$	Split products.	$\lim_{x \rightarrow 2} x(x+1)=6$
Quotient	$\lim(f/g)=(\lim f)/(\lim g)$, denominator limit $\neq 0$	Split quotients when valid.	$\lim_{x \rightarrow 2} (x^2+1)/x=5/2$
Factoring	Cancel common factor after factoring	Use for 0/0 polynomial limits.	$\lim_{x \rightarrow 3} [(x^2-9)/(x-3)]=6$
Rationalization	Multiply by conjugate	Use for radical 0/0 forms.	$\lim_{x \rightarrow 0} (\sqrt{x+1}-1)/x=1/2$
Trig limit	$\lim_{x \rightarrow 0} \sin(x)/x=1$	Use for basic trig limits.	$\lim_{x \rightarrow 0} \sin(3x)/x=3$

Continuity

Formula / Rule	Formula	Used for	Sample
Continuity at a	$f(a)$ exists; limit exists; $\text{limit}=f(a)$	Use to test continuity at a point.	$f(x)=x^2$ is continuous at $x=2$

III. Differential Calculus

Derivative rules

Formula / Rule	Formula	Used for	Sample
First principles	$f'(x)=\lim_{\Delta x \rightarrow 0} [(f(x+\Delta x)-f(x))/\Delta x]$	Use when asked for definition/ Δ method.	$f=x^2 \rightarrow f'=2x$
Constant	$d(c)/dx=0$	Derivative of a constant.	$d(15)/dx=0$
Identity	$d(x)/dx=1$	Derivative of x.	$d(x)/dx=1$
Power	$d(x^n)/dx=n x^{(n-1)}$	Main rule for powers.	$d(x^6)/dx=6x^5$
Constant multiple	$d(cu)/dx=c u'$	Pull constants outside.	$d(7x^4)=28x^3$
Sum/Difference	$(u \pm v)'=u' \pm v'$	Differentiate term-by-term.	$d(x^4+3x^2-7x)=4x^3+6x-7$
Product	$(uv)'=uv'+vu'$	Use for products of functions.	$d(x^2 \sin x)=2x \sin x+x^2 \cos x$
Quotient	$(u/v)'=(vu'-uv'')/v^2$	Use for quotients.	$d((x^2+1)/x)=(x^2-1)/x^2$
Chain	$d[f(g(x))]/dx=f'(g(x))g'(x)$	Use for a function inside another.	$d(3x^2+1)^5=30x(3x^2+1)^4$

Formula / Rule	Formula	Used for	Sample
Radical	$d(\sqrt{u})/dx = u'/(2\sqrt{u})$	Use for square-root functions.	$d(\sqrt{x^2+1}) = x/\sqrt{x^2+1}$

Trigonometric derivatives

Formula / Rule	Formula	Used for	Sample
Sine	$(\sin x)' = \cos x$	Differentiate sine.	$d(\sin 4x) = 4\cos 4x$
Cosine	$(\cos x)' = -\sin x$	Differentiate cosine.	$d(\cos 2x) = -2\sin 2x$
Tangent	$(\tan x)' = \sec^2 x$	Differentiate tangent.	$d(\tan 3x) = 3\sec^2 3x$
Cotangent	$(\cot x)' = -\csc^2 x$	Differentiate cotangent.	$d(\cot x) = -\csc^2 x$
Secant	$(\sec x)' = \sec x \tan x$	Differentiate secant.	$d(\sec x) = \sec x \tan x$
Cosecant	$(\csc x)' = -\csc x \cot x$	Differentiate cosecant.	$d(\csc x) = -\csc x \cot x$

Exponential/logarithmic derivatives

Formula / Rule	Formula	Used for	Sample
Exponential e	$(e^x)' = e^x$	Differentiate e-based exponentials.	$d(e^{3x}) = 3e^{3x}$
General exponential	$(a^x)' = a^x \ln a$	Differentiate a^x .	$d(5^x) = 5^x \ln 5$
Natural log	$(\ln u)' = u'/u$	Differentiate logarithms.	$d(\ln(x^2+1)) = 2x/(x^2+1)$

Inverse trig derivatives

Formula / Rule	Formula	Used for	Sample
Arcsine	$d(\arcsin u) = u'/\sqrt{1-u^2}$	Differentiate inverse sine.	$d(\arcsin x) = 1/\sqrt{1-x^2}$
Arccosine	$d(\arccos u) = -u'/\sqrt{1-u^2}$	Differentiate inverse cosine.	$d(\arccos x) = -1/\sqrt{1-x^2}$
Arctangent	$d(\arctan u) = u'/(1+u^2)$	Differentiate inverse tangent.	$d(\arctan(x^2)) = 2x/(1+x^4)$

IV. Applications of Derivatives

Implicit differentiation

Formula / Rule	Formula	Used for	Sample
Basic rule	$d(y^n)/dx = n y^{(n-1)} y'$	Use when x and y occur together.	$x^2+y^2=25 \rightarrow y' = -x/y$
Second implicit derivative	Differentiate y' again, applying product/chain rules	Use when y'' is requested.	For $x^2+y^2=25$, $y'' = -1/y - x^2/y^3$

Tangent & normal lines

Formula / Rule	Formula	Used for	Sample
Tangent slope	$m = f'(a)$	Slope of curve at $x=a$.	$y=x^2$ at $x=2 \rightarrow m=4$
Point-slope	$y-y_1 = m(x-x_1)$	Equation through a point with known slope.	$(2,4)$, $m=4 \rightarrow y-4=4(x-2)$
Normal slope	$m_n = -1/m_t$	Slope perpendicular to tangent.	If $m_t=4$, $m_n = -1/4$
Parallel	$m_1 = m_2$	Parallel lines have equal slopes.	$x+2y=6 \rightarrow m = -1/2$

Related rates

Formula / Rule	Formula	Used for	Sample
General method	Write relation $\rightarrow$ differentiate w.r.t. $t \rightarrow$ substitute values	Use when quantities change with time.	$A = \pi r^2$; $dr/dt = 3$ at $r=5 \rightarrow dA/dt = 30\pi$

Max/min

Formula / Rule	Formula	Used for	Sample
Critical number	$f'(x)=0$ or f' undefined (in domain)	Find candidates for extrema.	$f=x^2-4x+3 \rightarrow x=2$
Second derivative test	$f''(c)>0$ min; $f''(c)<0$ max	Classify a critical point when $f'(c)=0$.	$f=x^2-4x+3 \rightarrow f''=2>0$, minimum
First derivative test	$+$ to $-$ = max; $-$ to $+$ = min	Classify using sign changes.	For x^2-4x+3 : $-$ to $+$ at $x=2 \rightarrow$ minimum

Concavity & inflection

Formula / Rule	Formula	Used for	Sample
Concave up	$f''(x)>0$	Graph bends upward.	$f=x^2 \rightarrow f''=2>0$
Concave down	$f''(x)<0$	Graph bends downward.	$f=-x^2 \rightarrow f''=-2<0$
Inflection point	Concavity changes	Find candidates from $f''=0$ /undefined, then test change	$f=x^3 \rightarrow f''=6x$ changes sign at 0

Differentials & approximation

Formula / Rule	Formula	Used for	Sample
Differential	$dy=f'(x)dx$	Approximate small changes.	$y=x^2$, $x=5$, $dx=.01 \rightarrow dy=.1$
Linear approximation	$f(x+\Delta x)=f(x)+f'(x)\Delta x$	Approximate nearby values.	$\sqrt{4.04} \approx 2+(1/4)(.04)=2.01$

Curvature

Formula / Rule	Formula	Used for	Sample
Curvature	$K= y'' /[1+(y')^2]^{3/2}$	Measure how sharply a curve bends.	For $y=x^2$ at $x=0$: $K=2$
Radius of curvature	$\rho=[1+(y')^2]^{3/2}/ y'' $	Radius of the osculating circle.	For $y=x^2$ at $x=0$: $\rho=1/2$
Center of curvature	$X=x-p(1+p^2)/q$; $Y=y+(1+p^2)/q$, $p=y'$, $q=y''$	Locate the center of curvature.	For $y=x^2$ at $(0,0)$: $(0,1/2)$

V. Integral Calculus

Basic integration

Formula / Rule	Formula	Used for	Sample
Antiderivative	$\int f(x)dx=F(x)+C$ if $F'=f$	Reverse differentiation.	$\int 2x dx=x^2+C$
Power rule	$\int x^n dx=x^{n+1}/(n+1)+C$, $n \neq -1$	Integrate powers.	$\int x^4 dx=x^5/5+C$
Constant	$\int c dx=cx+C$	Integrate constants.	$\int 7 dx=7x+C$
Constant multiple	$\int c f dx=c \int f dx$	Pull constants outside.	$\int 5x^3 dx=5x^4/4+C$
$1/x$	$\int dx/x=\ln x +C$	Special case $n=-1$.	$\int 5/x dx=5\ln x +C$

Standard integrals

Formula / Rule	Formula	Used for	Sample
Exponential	$\int e^x dx=e^x+C$	Integrate e^x .	$\int e^{(3x)}dx=e^{(3x)}/3+C$
General exponential	$\int a^x dx=a^x/\ln(a)+C$	Integrate a^x .	$\int 2^x dx=2^x/\ln 2+C$
Sine	$\int \sin x dx=-\cos x+C$	Integrate sine.	$\int \sin x dx=-\cos x+C$
Cosine	$\int \cos x dx=\sin x+C$	Integrate cosine.	$\int \cos x dx=\sin x+C$
Secant squared	$\int \sec^2 x dx=\tan x+C$	Recognize derivative of tan.	$\int \sec^2 x dx=\tan x+C$
Cosecant squared	$\int \csc^2 x dx=-\cot x+C$	Recognize derivative of cot.	$\int \csc^2 x dx=-\cot x+C$
Secant tangent	$\int \sec x \tan x dx=\sec x+C$	Recognize derivative of sec.	$\int \sec x \tan x dx=\sec x+C$
Cosecant cotangent	$\int \csc x \cot x dx=-\csc x+C$	Recognize derivative of csc.	$\int \csc x \cot x dx=-\csc x+C$
Tangent	$\int \tan x dx=\ln \sec x +C$	Integrate tangent.	$\int \tan x dx=\ln \sec x +C$
Cotangent	$\int \cot x dx=\ln \sin x +C$	Integrate cotangent.	$\int \cot x dx=\ln \sin x +C$

Formula / Rule	Formula	Used for	Sample
Secant	$\int \sec x \, dx = \ln \sec x + \tan x + C$	Integrate secant.	$\int \sec x \, dx = \ln \sec x + \tan x + C$

Substitution

Formula / Rule	Formula	Used for	Sample
u-substitution	$u=g(x), du=g'(x)dx$	Use when an inner function and its derivative appear.	$\int (x^2+4)^5 dx = (x^2+4)^6/6 + C$
General power substitution	$u^n du = u^{(n+1)}/(n+1) + C$	Simplify composite integrals.	$\int 2x(x^2+1)^4 dx = (x^2+1)^5/5 + C$

Inverse-trig forms

Formula / Rule	Formula	Used for	Sample
Arctangent form	$\int dx/(a^2+x^2) = (1/a)\arctan(x/a) + C$	Recognize a^2+x^2 .	$\int dx/(9+x^2) = (1/3)\arctan(x/3) + C$
Arcsine form	$\int dx/\sqrt{a^2-x^2} = \arcsin(x/a) + C$	Recognize $\sqrt{a^2-x^2}$.	$\int dx/\sqrt{25-x^2} = \arcsin(x/5) + C$
Quadratic log form	$\int dx/(x^2-a^2) = (1/2a)\ln (x-a)/(x+a) + C$	Integrate difference of squares.	$\int dx/(x^2-9) = (1/6)\ln (x-3)/(x+3) + C$
$\sqrt{x^2+a^2}$ form	$\int dx/\sqrt{x^2+a^2} = \ln x+\sqrt{x^2+a^2} + C$	Recognize quadratic radical.	$\int dx/\sqrt{x^2+4} = \ln x+\sqrt{x^2+4} + C$

Integration by parts

Formula / Rule	Formula	Used for	Sample
Formula	$\int u \, dv = uv - \int v \, du$	Use for products where substitution is not enough.	$\int x e^x \, dx = e^x(x-1) + C$

Definite integrals

Formula / Rule	Formula	Used for	Sample
Fundamental Theorem	$\int_a^b f(x) dx = F(b) - F(a)$	Evaluate definite integrals.	$\int_0^2 x^2 dx = 8/3$
Area under curve	$A = \int_a^b f(x) dx$ (when $f \geq 0$)	Find geometric area above x-axis.	$\int_0^1 x \, dx = 1/2$
Area between curves	$A = \int_a^b (\text{top} - \text{bottom}) dx$	Find area between graphs.	Between x and x^2 on $[0,1]$: $1/6$

Applications

Formula / Rule	Formula	Used for	Sample
Disk method	$V = \pi \int_a^b R^2 \, dx$	Volume of revolution with no hole.	Rotate $y=x$, $0..1$: $V = \pi/3$
Washer method	$V = \pi \int_a^b (R^2 - r^2) dx$	Volume with a hole.	Use outer radius minus inner radius squared
Shell method	$V = 2\pi \int (\text{radius})(\text{height}) dx$	Volume using cylindrical shells.	About y-axis: $V = 2\pi \int x f(x) dx$
Arc length	$L = \int_a^b \sqrt{1+(y')^2} \, dx$	Length of a curve.	For $y=x^2$ on $[0,1]$: $\int \sqrt{1+4x^2} dx$
Surface area x-axis	$S = 2\pi \int_a^b y \sqrt{1+(y')^2} \, dx$	Surface generated around x-axis.	Apply to a specified $y=f(x)$
Surface area y-axis	$S = 2\pi \int_a^b x \sqrt{1+(y')^2} \, dx$	Surface generated around y-axis.	Apply to a specified $y=f(x)$

VI. Trigonometry, Parametric & Polar

Trig identities

Formula / Rule	Formula	Used for	Sample
Pythagorean	$\sin^2 x + \cos^2 x = 1$	Simplify trig expressions/integrals.	$\sin^2 x = 1 - \cos^2 x$
Pythagorean	$1 + \tan^2 x = \sec^2 x$	Simplify tan/sec expressions.	$\sec^2 x - 1 = \tan^2 x$
Pythagorean	$1 + \cot^2 x = \csc^2 x$	Simplify cot/csc expressions.	$\csc^2 x - 1 = \cot^2 x$
Double angle	$\sin 2x = 2 \sin x \cos x$	Convert products/powers.	$\sin x \cos x = (1/2)\sin 2x$
Power reduction	$\sin^2 x = (1 - \cos 2x)/2$	Integrate even powers of sine.	$\int \sin^2 x \, dx = x/2 - \sin 2x/4 + C$
Power reduction	$\cos^2 x = (1 + \cos 2x)/2$	Integrate even powers of cosine.	$\int \cos^2 x \, dx = x/2 + \sin 2x/4 + C$

Parametric

Formula / Rule	Formula	Used for	Sample
First derivative	$dy/dx=(dy/dt)/(dx/dt)$	Find slope from parametric equations.	$x=t^2, y=t^3 \rightarrow dy/dx=3t/2$
Second derivative	$d^2y/dx^2=[d/dt(dy/dx)]/(dx/dt)$	Find second derivative parametrically.	Apply to previous example

Polar

Formula / Rule	Formula	Used for	Sample
Conversion	$x=r \cos\theta, y=r \sin\theta$	Convert polar to Cartesian.	$r=2, \theta=\pi/3 \rightarrow (1, \sqrt{3})$
Relation	$x^2+y^2=r^2$	Convert circle/radial expressions.	$x^2+y^2=9 \rightarrow r=3$
Polar area	$A=(1/2)\int_{\alpha}^{\beta} r^2 d\theta$	Find area in polar coordinates.	For $r=2, 0..\pi$: $A=2\pi$

VII. Geometry formulas used in calculus

Circle

Formula / Rule	Formula	Used for	Sample
Area	$A=\pi r^2$	Circle area.	$r=3 \rightarrow A=9\pi$
Circumference	$C=2\pi r$	Circle circumference.	$r=3 \rightarrow C=6\pi$

Common solids

Formula / Rule	Formula	Used for	Sample
Sphere volume	$V=(4/3)\pi r^3$	Sphere volume.	$r=3 \rightarrow V=36\pi$
Sphere surface	$S=4\pi r^2$	Sphere surface area.	$r=3 \rightarrow S=36\pi$
Cylinder	$V=\pi r^2 h$	Cylinder volume.	$r=2, h=5 \rightarrow 20\pi$
Cone	$V=(1/3)\pi r^2 h$	Cone volume.	$r=3, h=4 \rightarrow 12\pi$

Coordinate geometry

Formula / Rule	Formula	Used for	Sample
Slope	$m=(y_2-y_1)/(x_2-x_1)$	Slope between two points.	$(1,2),(3,6) \rightarrow m=2$
Distance	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance between points.	$(0,0),(3,4) \rightarrow d=5$
Pythagorean	$a^2+b^2=c^2$	Right-triangle relationships; common in related rates.	$3^2+4^2=5^2$

Quick 'Which Formula Do I Use?' Guide

Limit: Try direct substitution. If $0/0$, try factoring, rationalizing, or trig identities.

Derivative: Power $\rightarrow$ sum/difference $\rightarrow$ product/quotient $\rightarrow$ chain; use implicit differentiation if x and y are mixed.

Tangent: Find y' , evaluate at the point for m , then use $y - y_1 = m(x - x_1)$.

Normal: Use $m_{\text{normal}} = -1/m_{\text{tangent}}$.

Related rates: Write the relationship, differentiate with respect to time, substitute, solve.

Maximum/minimum: Find critical numbers from $f' = 0$ /undefined, then use first or second derivative tests.

Concavity: Use f'' ; positive means concave up, negative means concave down.

Basic integration: Power rule unless exponent is -1 ; then use $\ln|x|$.

u-substitution: Look for an inner function together with its derivative.

Integration by parts: Try for products such as $x e^x$, $x \ln x$, $x \sin x$.

Definite integral: Find an antiderivative and use $F(b) - F(a)$.

Area between curves: Integrate top minus bottom.

Volume: Disk: $\pi \int R^2 dx$; washer: $\pi \int (R^2 - r^2) dx$; shell: $2\pi \int \text{radius} \cdot \text{height} dx$.

Core formulas to memorize first

- Power rule: $d(x^n)/dx = n x^{(n-1)}$
- Product rule: $(uv)' = uv' + vu'$
- Quotient rule: $(u/v)' = (vu' - uv')/v^2$
- Chain rule: $d[f(g(x))]/dx = f'(g(x))g'(x)$
- Derivative: $(\sin x)' = \cos x$; $(\cos x)' = -\sin x$; $(\tan x)' = \sec^2 x$
- Derivative: $(e^x)' = e^x$; $(\ln x)' = 1/x$
- Power integral: $\int x^n dx = x^{(n+1)}/(n+1) + C$, $n \neq -1$
- Special integral: $\int dx/x = \ln|x| + C$
- Substitution: $u = g(x)$, $du = g'(x)dx$
- Parts: $\int u dv = uv - \int v du$
- FTC: $\int_a^b f(x)dx = F(b) - F(a)$
- Curvature: $K = |y''|/[1 + (y')^2]^{3/2}$