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Rethinking what is valuable in mathematics and statistics education

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ABSTRACT
The current rapid development of generative AI is creating an existential crisis in mathematics and statistics education similar to the one in which the electronic calculator replaced the slide rule and logarithm tables. If everything that we teach in undergraduate mathematics and statistics can be done by the average smartphone, then why are we teaching it? As a part of a larger study into educators' opinions on how mathematics and statistics should be taught and assessed, we asked them what students should gain from their studies in the subject. In this paper we use thematic analysis to analyse the responses. Four primary themes emerged from the interview data, namely content mastery, mathematical thinking, mathematical communication and appreciation of mathematics and statistics. Content mastery was divided into two sub-themes of computational mastery and conceptual understanding. These are discussed in light of the development of generative AI and the current practices of teaching mathematics and statistics. We suggest a refocusing of mathematics and statistics education to place more explicit emphasis on the conceptual and higher thinking aspects of courses and begin a discussion of how this could be achieved within the time constraints faced by educators.

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1. Introduction

Computing systems are making rapid gains in mathematical capability. Since the conception of computational and algebraic mathematics computing systems (such as Maple, Mathematica and Matlab) in the 1980s, many such systems have been developed, and improvements in computing power and algorithms have allowed the creation of increasingly sophisticated solution systems. These systems are able to provide correct worked solutions to computational problems more quickly and more accurately than humans. In Vaswani, 2017, Vaswani et al. published a paper with a new network architecture for machine learning that paved the way for the current generation of generative AI systems.

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Specialised AI systems can now solve mathematics problems that involve higher order and creative reasoning. In 2024 Google announced that an AI combination of their AlphaProof formal reasoning machine and their AlphaGeometry 2 geometric reasoning machine solved four of the six problems on the 2024 International Mathematical Olympiad paper, equating to a silver medal performance, although not within the examination time limits (GoogleDeepMind, 2024; Roberts, 2024). In 2025, DeepSeek R1 was tested on a set of difficult third year mathematics problems. Not only did the machine quickly and correctly solve the problems but it was also able to explain its reasoning and discuss the 'thinking' that it did to arrive at the solution, including the dead-end pathways that it tried (personal correspondence).

The advent of such systems is creating much discussion about the ways in which 'generative AI will transform knowledge work' (AbuMusab, 2024; Alavi & Westerman, 2023; Lee et al., 2025; Storey et al., 2025) and claims about the skills that will be rendered obsolete and those that will still be important (AbuMusab, 2024; Cole et al., 2021; Höög & Ljungqvist, 2024; McKendrick, 2023). There is a need for humans to develop the skills to use generative AI effectively, and to develop skills to do things that machines cannot. Skills reported to be of increasing value in an AI-saturated world include critical thinking, problem-solving, collaboration, creativity, communication, adaptability and the ability to be a lifelong learner (Cole et al., 2021; Dolev & Itzkovich, 2020; Hutson & Ceballos, 2023; Sousa & Wilks, 2018).

On the one hand, mathematics and statistics (M&S) can be seen as central to the whole area of AI. Machine learning algorithms and models are underpinned by M&S (e.g. Bender, 1996; Singh, 2023), and so expertise in M&S enables understanding of the capabilities and limitations of machines. Additionally, it has been posited that 'mathematical intelligence' is one of the things that sets humans apart from machines and allows them to do things that machines cannot (Mubeen, 2022).

On the other hand, beliefs that M&S is about putting numbers into formulas and doing calculations are widespread amongst students (Crawford et al., 1994; Schaathun, 2022), and views that M&S are difficult, boring, and dislikable are prevalent in the wider Western society (Andrade-Molina & Montecino, 2024; Itter & Meyers, 2017; Russo et al., 2023). Possibly these perceptions also stem from a belief that M&S is about carrying out computations, an activity that is best done by machines and not humans.

These social and technological changes and conflicting perceptions lead to questions about M&S education: what exactly should be taught to students, and in what ways, in order for them to flourish throughout their futures? The advent of AI will necessarily disrupt M&S education in a similar, or even greater way than the introduction of electronic pocket calculators, and educators need to reflect on, and rethink the core aims of M&S education to ensure its ongoing relevance.

A traditional undergraduate university mathematics experience begins with the study of calculus and linear algebra – learning, amongst other things, techniques for differentiation, integration and matrix manipulation, with an emphasis on being able to carry out these computations by hand. Introductory statistics programmes typically begin with procedures for descriptive statistics and hypothesis tests, and students are usually taught how to use computer tools to carry out these procedures. These types of courses provide the foundation from which students can go on to study more advanced M&S, and can also apply these techniques within the context of other disciplines such as physics, engineering and business.

However, given that machines can now do all of these mathematical computations more quickly and more accurately than humans, what is the ongoing purpose of this instruction? Is teaching students proficiency in finding exact solutions to second order differential equations by hand still a useful skill? To what extent should mathematics education shift to focus on learning to use machines to carry out tasks?

Statistics education has already undergone such a shift (Gomez, 2014; Zavez & Harel, 2025), with computations routinely being carried out using computers. Yet the same type of questions can still be asked. For example, how much of the detail of hypothesis testing do students need to know? Do they need to know how to choose an appropriate underlying distribution? Do they need to know the formulas for the calculations? Do they need to know how to calculate a p-value, or is knowing what it means sufficient?

Taking this line of questioning even further: since machines are reaching the point where they can solve problems that involve reasoning, to what extent can M&S simply be outsourced to AI? Would it be better to teach students how to use machines to answer questions (using a critical, intelligent approach, not an approach of blind acceptance), and how to evaluate work (both human and AI-generated) rather than to spend precious time teaching them to do things that machines can do? Or is there intrinsic value in understanding the details of these procedures, and being able to do them for oneself?

These are important questions that affect both the conceptualisation and the delivery of M&S education. As such, these questions provoke strongly held, diverse responses from M&S educators.

Before considering such questions, it is useful to first step back and examine the purpose of the M&S education being provided. Universities offer a wide range of units of study in M&S, from foundational units that cover high school level mathematics, to service subjects for other disciplines such as statistics for business students or tensor analysis for engineering students, through to advanced units for senior M&S students. Different units have different stated outcomes. For example, a unit in nursing numeracy may aim for mastery of numerical computations (such as unit conversions), while a unit in algebraic topology might focus on deep conceptual understanding of topological spaces and their invariants, and on aspects of proof. However, beyond the specifics of individual units of study, are there knowledge and skills that are widely perceived as a core part of learning M&S that may be of value in debates about what students should learn and how they should be taught?

As part of a study investigating perceptions of educators on the teaching and learning of mathematics, statistics and numeracy, we interviewed M&S educators to ascertain their views on what they believed to be the purposes of teaching the unit(s) of study with which they were involved.

2. Purpose of the study

In this paper, we pose the following research questions:

- What knowledge and skills are regarded by M&S educators as important for all students to develop, regardless of the unit they are studying?
- In what ways do these views align and/or diverge with the realities of an AI-rich world?
- If points of conflict exist, how might M&S education adapt in response?

3. Methodology

M&S educators have been invited to participate in this study, which is ongoing. Recruitment has been by means of advertising at M&S education conferences, advertising through M&S community of practice networks, and word of mouth. Participants take part in a structured interview in which they are asked a number of questions, including what units of study they teach, and what they believe that students should gain from studying those units. The precise question asked which relates to this study is ‘What do you think that students should gain from this subject?’

Units of study which students enrol in for one teaching period are referred to by different names at different universities. At Western Sydney University, they are referred to as ‘subjects’ and hence this language was used in the question. However, the meaning of the term has been clarified with interviewees during the interview when necessary.

Ethics approval was provided by Western Sydney University, approval number H15854.

The results of the first thirty interviews were transcribed and then analysed using thematic analysis. Thematic analysis is a method for identifying and interpreting patterns of meaning across qualitative data in which interview transcripts are reviewed in detail and segments of text are coded to capture key ideas. These codes are then examined and grouped into themes that reflect broader, meaningful patterns relevant to the research question. (See Braun & Clarke, 2012 and Mullen et al. (2021) for a more thorough treatment.) The data were also examined for trends as a function of educator geographical location, educator age and experience, and the type of unit being taught.

4. Educator and subject profiles

To date, 33 mathematics and statistics educators from 17 universities across Australia, New Zealand, Finland, Ireland, the UK and the USA participated in the study. Each participant was involved in teaching mathematics or statistics to undergraduate students from a range of disciplines, including mathematics, physics, engineering, biology, business, education and nursing. Many participants taught multiple units. These educators chose to discuss 1–3 units in the interview. This allowed specific, in-depth reflection on what specific student cohorts should gain from studying specific units. 18 of the educators taught only mathematics units, 5 taught only statistics units and 10 taught both mathematics and statistics units.

Table 1 lists different areas of study that were discussed in the interviews.

5. Thematic analysis

The interviews generated a wide range of thoughts and ideas from the educators. Each response was summarised and grouped with similar ideas, and the most frequently mentioned were identified as key themes. Theme titles were then selected to reflect the central concepts they represented.

The main themes to emerge were remarkably consistent across educators, and independent of educator location, age or experience. These themes were seen as central to the education of both mathematics and statistics students. They also remained stable across year of study, with educators wanting students to develop and refine the same skills and

Table 1. Topics of study discussed by educators during interviews. Some educators taught multiple units, both within and across topic areas. In many cases, educators had experience teaching additional units beyond those they chose to discuss in detail.

First year	Number of distinct educators
Calculus, vector calculus, linear algebra, differential equations	17
Statistics	4
Mathematics for nursing	1
Discrete mathematics, proof	6
Mathematics for education students	1
Foundational mathematics	9
Second year	
Calculus, linear algebra, differential equations, analysis	3
Statistics	2
Third year and higher	
Pure mathematics	3
Statistics	2
Analysis	2

expertise as they progressed through their degrees, rather than wanting the students to gain something different from more senior subjects.

Four primary themes emerged from the data. These were content mastery, mathematical thinking, mathematical communication, and appreciation of mathematics. The theme of content mastery was broken down into two sub-themes, being computational skills and conceptual understanding.

As educators reflected on what their students should gain, the interconnectedness of these themes became evident. For example, educators discussed the need for technical mastery in order to facilitate conceptual understanding, the importance of developing mathematical communication skills to deepen understanding, and of conceptual understanding leading to enjoyment and appreciation of M&S. Each of the themes is discussed in more detail in the following sections, and overlaps and tensions between themes are also noted.

6. Emergent themes

6.1. Content mastery

The primary theme to emerge, which was raised by every educator, was that students should gain knowledge of, and mastery of, the subject matter being taught. This is something of a truism, as the commonly understood purpose of teaching content is so that students learn it (Hirst, 1971), and the role of the teacher is to optimise the student learning experience (Higginson, 1980). Delving deeper into exactly what was meant by student learning, as well as the reasons why the educator felt that these learnings were important, yielded more interesting and more nuanced results. These were expressed in two main subthemes: mastery of computational skills and conceptual understanding.

This theme aligns fully with existing cognitive perspectives on learning mathematics, which define three components of mathematical knowledge: declarative, procedural and conceptual. Declarative knowledge is a set of memorised facts that 'can be conceptualized as a network of relationships containing basic problems and their answers, such as

$4 + 7 = 11$ or $11 - 4 = 7$' (Goldman & Hasselbring, 1997). Procedural knowledge is the set of algorithms used to solve tasks. It is 'represented as step-by-step instructions in how to complete tasks, and the steps are to be executed in a predetermined linear sequence'. (Goldman & Hasselbring, 1997). Procedural knowledge involves the application of declarative knowledge. Definitions of conceptual knowledge are often vague or poorly operationalised (Crooks & Alibali, 2014), however, it can be conceptualised 'as a connected web of information in which the linking relationships are as important as the pieces of discrete information that are linked'. (Goldman & Hasselbring, 1997) that determines understanding (Hiebert & Lefevre, 1986). Declarative and procedural knowledge are embedded within the subtheme of computational skills, while conceptual knowledge is encompassed by the subtheme of conceptual understanding.

Seventeen of the educators spoke explicitly of computational or procedural skills as being an important learning outcome for their units. All gave the reason that these skills would be needed in future subjects or careers, or in the case of service subjects, because they were specified as required skills by the students' discipline. For example, engineering students needed to master methods of integration in first year calculus because this was required knowledge for second year calculus, and business students needed to learn to carry out hypothesis tests on different data sets because this was required in later business units.

> It's more about, I guess, the techniques and we're exposing them to all sorts of things that would be useful in IT (Ed 19)

> It's heavily skills based. And we know this because … the engineers over the years have given us guidance about the kinds of skills that they want their students to learn in that subject. (Ed 14)

> They should have a solid understanding of both procedures and the underlying concepts. (Ed 11)

Many educators, particularly those teaching foundational classes, felt that it was essential that students gain competence in a basic set of computational skills such as the ability to manipulate algebra and the ability to work with fractions by hand.

> I think they should get technical competency - mastery of algebraic manipulation and other basic calculation skills so that when they go on to future maths and engineering subjects, they've got the tools to do what they need to do. (Ed 5)

Here, there was a slight difference in nuance between the views of mathematics and statistics educators. Mathematics educators who felt that procedural competence was an essential skill tended to emphasise the need for students to gain competence carrying out basic procedures by hand, whereas the statistics educators were focused on ensuring that students knew how to use software tools to carry out the procedures.

Although most educators explicitly indicated that they viewed conceptual understanding as being at least as important as computational skills, only a few educators felt that mastery of computational skills was not an important part of student learning in their units. These mathematics educators viewed the calculations primarily as a tool through which to develop conceptual understandings.

> It's not so important that they can do those calculations. It's knowing why those things are there and their connection to, if you like, quote unquote, real world phenomena. (Ed 09)

> If they're seeing maths as kind of a glorified stage show: just move these things, make some - rote learn how to reconfigure an equation, get an answer that gets you a tick - they're very capable of doing that, but I think that's fairly useless as an educational outcome. And I think what's actually going to stick with them longer term is actually resonating with what they're learning and actually seeing it in the world around them. (Ed 26)

Here again the statistics educators' responses tended to be nuanced slightly differently from those of the mathematics educators. These statistics educators also wanted their students to gain the necessary procedural skills, but generally only within a broader context of understanding and interpreting contextualised problems. That is, their descriptions of desired learning prioritised big-picture understanding, and the ability to select valid procedures.

> So it's really important students to have a grasp of: 'It's not how to do it, but what am I doing and why?' … Recipe book kind of, 'Do this, do this, do this' in statistics could be useful to outline to you how to use this software, but you need to understand what you are doing - whether it makes sense. (Ed 33)

The second sub-theme that emerged was that students should gain a deep conceptual understanding of the subject matter. Educators indicated that students should understand what they are doing, why they are doing it, and the situations in which it applies.

> It's about the concepts … trying to get them to understand the concepts (Ed 19)

> And especially what I want from them is an understanding of why things are the way they are. (Ed 30)

> So even though in 10 years' time they may have forgotten some of the technical details they know that there is some mathematics there that may well be relevant to what they need and they know how to go and look it up and at one stage they actually understood this stuff. (Ed 10)

> Students really thoroughly understanding the concepts concerned … really thoroughly, deeply in your bones getting the concepts. (Ed 17)

Educators also wanted their students to understand how the content they are learning fits into the broader picture of mathematical knowledge.

> Actually thinking about why things work instead of just being able to problem solve known problems is really what maths is all about. … So making those connections through the topics, too, is really important. (Ed 22)

A number of educators discussed a tension between teaching students computational skills and conceptual understanding. They felt that, due to time constraints and the quantity of material required to be covered, there was not time for students to learn both and conceptual understanding was not being adequately addressed.

> They don't gain a deep understanding of those areas because there is way too much material to develop understanding as well as just basic skills and facility with material. (Ed 1)

The literature offers no consensus on how the interrelationship between computational skills and conceptual understanding should be conceptualised. One perspective maintains that computational skills provide a necessary foundation for the development of conceptual understanding. This view is supported by studies such as Lauritzen (2012) and Mills (2016), who identified factual and procedural knowledge as essential dimensions of conceptual understanding, and by related research demonstrating that students who begin courses with strong foundational mathematics skills tend to outperform their peers (Intepe et al., 2019; Johnson & Kuennen, 2006; Rylands & Coady, 2009).

In contrast, some researchers have argued that high-level conceptual understanding should precede the development of procedural skills (e.g. Carpenter, 1986). Others suggest that skills and understanding develop in tandem through an iterative process (Rittle-Johnson et al., 2001). Yet another perspective proposes that 'it is the relationships among, and not the distinctions between, elements of procedural and conceptual knowledge that ought to be of primary interest' (Silver, 1986). There is, however, agreement that both procedural skills and conceptual knowledge are necessary for effective problem-solving (Al-Mutawah et al., 2019; Derequito et al., 2025; Lauritzen, 2012)

In this study, the interviews revealed a difference in how mathematics and statistics educators typically conceptualised the relationship between computational skills and conceptual understanding. Although both M&S educators viewed the acquisition of skills and understanding as an iterative process in which pre-existing knowledge and skills allow students to build further skills and understandings, many mathematics educators (though not all) regarded computational skills as a necessary foundation for developing conceptual understanding. In contrast, many of the statistics educators viewed computational skills as tools used to implement the solution to a problem that had already been conceptually understood.

6.2. Mathematical thinking

Mathematical thinking eludes simple definition, and there is no consensus of 'what it is' amongst theoreticians from across psychometric, cognitive-information processing, cultural, educational and mathematical perspectives (Sternberg, 1996). Moreover, what constitutes mathematical thinking seems to be contextual; for example, different processes seem to be required to solve a problem in algebraic group theory compared to solving a problem in an engineering control system. Sternberg (1996) suggests that mathematical thinking is possibly best modelled as a set of prototypes in the analytic, creative, practical and affective domains. It involves skills such as identifying patterns, making connections, and developing new ways to describe and understand situations. It encompasses many dimensions, such as: mathematizing problems; constructing appropriate solution models; having a range of problem-solving strategies available for use; being able to think through a problem in a logical, structured and rigorous manner; being able to generalise and abstract a finding; being able to synthesise knowledge from different learning topics to address a new problem; being able to creatively apply techniques to solve a problem; and having an appreciation of the power of mathematics and a sense of one's self-efficacy.

Many educators stated that developing mathematical thinking and problem-solving skills is something that they wish students to gain from their study. In this study we included notions of higher order thinking, methodology selection, critical thinking skills, critique of solutions, proof and problem-solving skills as part of the mathematical thinking theme. Our theme of mathematical thinking also incorporated concepts of rigorous and logical thinking:

> how to think mathematically - we often say think critically - but I sort of think about really sort of how to structure arguments and how to consider counter examples and sort of investigate whether statements are true in that sense. (Ed 30)

Mathematical thinking was widely viewed as an intrinsic component of mathematics, with one educator saying that they wish for their students to 'take the first steps in understanding mathematical thinking, what mathematics is' (Ed 24) and another describing mathematical thinking as being 'how to do mathematics' (Ed 14).

> I guess the most important things that the students should learn there, besides the elementary skills, is to learn a particular way of thinking. To learn how to solve problems, how to chop a problem into small parts, which they can solve, perhaps in a small group or so. Critical thinking skills. So I like to think that we use mathematics throughout the degree as some sort of a scaffold for the students to learn their problem solving skills. (Ed 6)

> Really the content is less relevant than sort of the approach to mathematics. So we're trying to get them to understand logic, to understand proof. (Ed 19)

> How to think mathematically - we often say think critically - but I sort of think about really sort of how to structure arguments and how to consider counter examples and sort of investigate whether statements are true in that sense. (Ed 30)

Interviewed educators acknowledged the many students who hold a mindset that mathematics is about rote-learned procedures, and the need to counter this view:

> [There are] students who basically rote learn everything.... They've just rote learned how to; the methods the high school teacher taught them. They went away and practiced that a hundred times and now they're good at it, but they have no idea what they're doing and zero ability to apply it out of that immediate context and that immediate framing, which to me is a disaster. (Ed 26)

Mathematical thinking is the opposite of performing rote-learned procedures and so developing mathematical thinking skills is essential to allow meaningful learning (Novak, 2002; Oliveira, 2023).

6.3. Mathematical communication

Developing skills in mathematical communication is widely recognised as a fundamental component of learning mathematics and statistics (Morgan et al., 2014). Learning in these disciplines involves both engaging with existing knowledge and methods, and constructing one's own solutions to problems – activities that rely heavily on effective communication. The National Council of Teachers of Mathematics (1989, p. 214) emphasised that students should develop both receptive and expressive communication skills in their mathematics and statistics studies.

Receptive language skills enable students to understand, interpret, and critically evaluate mathematical ideas presented in written, oral, or visual forms. These skills are essential for accessing and making sense of existing knowledge in mathematics and statistics. Expressive language skills, on the other hand, allow students to articulate mathematical ideas through speaking, writing, demonstrating, and using visual representations. They also involve using appropriate mathematical vocabulary, notation, and structures to represent ideas, describe relationships, and model situations.

Such expressive communication skills are vital not only for responding to existing mathematical knowledge but also for developing original work. Moreover, expressive communication has been shown to deepen students' conceptual understanding, enhance their mathematical performance, and reduce anxiety towards mathematics (Lomibao et al., 2016).

In this study we classified ideas about receptive and expressive communication, and also peer collaboration under the theme of mathematical communication. Developing proficiency in mathematical communication was raised as a very important learning outcome by many of the educators. A number of educators observed that being able to solve a problem yourself is of little benefit if you cannot share your solution with others.

> I think a fundamental thing that they should get out of these units is the ability to express technical information. I mean, almost no one's going to get by on their sheer technical brilliance. They're going to be involved in explaining technical ideas and conceptions and calculations to other people, and they have to be able to explain. And for me, the communication side of things and the explanation are hugely important and not given the attention and the time that they need. (Ed 9)

Educators felt that students should learn to read written mathematics, and to learn to distinguish between good and bad mathematics. They should also learn to write correct, clear mathematics themselves. They should learn to explain mathematics to others.

> So be able to read a piece of mathematics and evaluate its quality and also be able to communicate mathematics effectively, correctly and effectively using correct notation. I think they should learn to do that as well. They should be able to explain their mathematical reasoning and solutions to their peers. They should be able to distinguish between a good mathematical solution and a bad mathematical solution. (Ed 5)

> How can you explain it to your grandmother, assuming your grandmother is not a statistician.… It's really important because giving your grandmother a two pages computer output or writing one page equations means absolutely nothing.… So you need to be able to write one paragraph that could be read by anyone and everyone and still understood. (Ed 33)

In order to be able to do so, it is first necessary to have a sound understanding of the concepts involved, and so the act of communicating mathematically facilitates deeper understanding.

> I think students should be developing confidence in not only mathematical writing but mathematical communication more broadly. And what underlies that is students really thoroughly understanding the concepts concerned such that they can articulate those concepts in ways that are meaningful beyond the kind of esoteric abstract language that is often used in kind of mathematical education and research circles. (Ed 17)

A number of educators also mentioned the ability to do mathematics collaboratively, which involves both oral and written mathematical communication, as being important.

> I want them to learn to read and write mathematics and learn to collaborate with other people. (Ed 24)

> When they are communicating with their peers and [saying] 'I don't know how to do this' and one of them is trying - not just writing on the board - like, trying to discuss with them; I feel they get it - like, get it faster - because they feel all right.… Having that peer help is really beneficial and it adds to their learning. (Ed 4)

6.4. Appreciation of the discipline

The fourth theme that was identified from the interviews was the desire that students develop an appreciation of M&S through their studies. Appreciating M&S involves being attuned to its presence and relevance. It reflects a person's ability to recognise mathematical aspects in everyday situations and a willingness to apply mathematical thinking when suitable. This appreciation grows through the study of mathematics and is nurtured by engaging and rewarding learning experiences (Walter, 1963). A number of different aspects of appreciation were raised by educators, with different aspects being raised by different educators.

Some educators wanted students to appreciate the utility and relevance of M&S.

> They should come away, I think, with an appreciation of the power of mathematics and its applications. Even though I'm a pure mathematician but I always try to have in my kit bag a couple of applications for each little piece of maths that I'm going to talk about because I want students to realise that mathematics is incredibly useful in the real world. (Ed 10)

A number of statistics educators emulated real-world practice in the classroom, which assisted development of mathematical appreciation.

> And then I introduced group projects as well [in which] they worked on live data with different companies … And then I noticed a big turnaround in, well, firstly, enthusiasm because I don't think there'd ever been any positive feedback about the stats modules ever - before I started doing them. And then, you know, once I introduced those, some students were saying it was their favourite subject. (Ed 32)

> I think a lot of stories in the media these days are data stories because that's where journalists pick up interesting facts and figures from research, from science, from political surveys, and then they write a story about it. … And so it's making sense of that, giving them the skills to dig into the data, making them aware of how they can find data themselves and write their own data story. (Ed 14)

Other mathematics educators wanted students to learn about, and appreciate, the social history of mathematics and its nature as a living, evolving discipline.

> And so what I'd like the students to get out of it when they get to the end of the subject is: Maths [is] a living, evolving subject. (Ed 22)

> That mathematics existed for a long time, there's a long history of it, that it's sort of used in other disciplines, that it's used in workplaces and that mathematical knowledge maybe stands a little bit separate to other sort of disciplines of knowledge and knowledge creation. (Ed 28)

> But certainly I think we're doing a disservice to them if we don't give them some sort of concept of mathematics is like an organic subject (Ed 30)

Still others highlighted the interrelationships within M&S, and between M&S and other disciplines of study, and wanted students to appreciate this interconnectedness.

> 'I say, 'Well, you know, we're doing vectors here and it's a topic, but don't forget about it because it'll come back to you' … And so, you know, you can bring up things that … 'Remember this idea that we had? Here it is again. It looks a little bit different but it's exactly the same as the idea we had before.' So making those connections through the topics, too, is really Important. (Ed 22)

Finally, there were educators who wanted students to gain enjoyment from studying the unit.

> So that's one of the things I want them to have, an appreciation of the power of mathematics and what an interesting subject it is. (Ed 10)

6.5. Other objectives raised by educators

There was a notable consistency in what the interviewed educators desired for their students. There were, however, several objectives that were raised by some educators but which did not come through strongly enough to be classified as a main theme, because they were tied to particular subsets of subjects. A number of educators of first year units that were taken mostly by school leavers mentioned that one of the aims of their unit was to change students' conceptualisations of mathematics from procedural-based to a discipline about higher order, more abstract thinking, with a focus on communicating ideas and reasoning.

> It has a bit of a transition from high school to university flavour to it … it's sort of changing the mindset of students so that they sort of start to see mathematics in perhaps a different way. (Ed 20)

> Ideally they should also learn about what maths is at the university level as opposed to the high school level, which means more in depth, more explanations and much better communication skills in mathematics and mathematical reasoning. (Ed 31)

Some of these first year educators also sought to build a sense of belonging in their students.

> My first thought was integration to university. So they are first year students; I want them to feel that they belong and that they are part of the community of mathematics learners. (Ed 24)

Many of the educators teaching foundation subjects wanted their students to gain self-confidence in their M&S capabilities and self-efficacy in the discipline.

> I like to think that I'm more trying also to build self-confidence in mathematics and self-efficacy in mathematics and statistics so that they, you know, develop the skills that they need to continue to succeed in their chosen careers. (Ed 7)

7. Discussion

7.1. Connections between themes

The themes identified through the interviews are interrelated and interdependent, as illustrated in Figure 1. Black arrows represent relationships we deem necessary, while blue arrows indicate connections we consider beneficial but not essential. For instance, content mastery is necessary for mathematical thinking, which emerges from detailed engagement with mathematical content, but it is possible to (superficially) master content without mathematical thinking, although such thinking certainly enhances content mastery. Receptive language skills are required for content mastery, which in turn supports expressive language. Understanding some content is necessary for mathematical appreciation, though content can be learned without appreciating it. M&S cannot be appreciated without being able to read and understand it, yet it is possible to communicate mathematically without any such appreciation. We also suggest that mathematical appreciation may exist without mathematical thinking – analogous to non-musicians appreciating a Beethoven symphony – though deeper knowledge enhances appreciation.

Assigning black or blue arrows was a subjective judgment. For example, the arrow from expressive to receptive language was coloured black to reflect the necessity of active engagement for advanced comprehension; however, given the typical developmental sequence from receptive to expressive language, it could be argued that the connection is beneficial but not necessary.

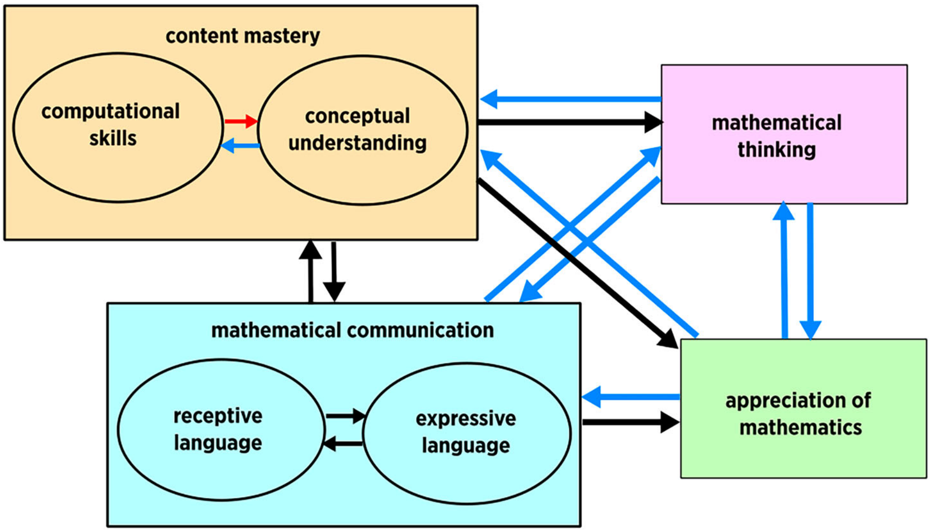

Figure 1. Relationships between the themes. Black arrows indicate directional relationships that we believe are necessary, while blue arrows indicate relationships that we think are beneficial but not necessary. The red arrow indicates a connection we discuss in greater depth.

There is one connection that we believe warrants more careful consideration. This is the red-coloured connection from computational skills to conceptual understanding, and it is discussed in more depth later in this paper.

7.2. Valuable skills in an AI world

Conceptual understanding, mathematical thinking and mathematical communication were raised by interviewees as important educational outcomes for M&S units. As discussed in the introduction, these are all higher order skills that are perceived to be desirable in an AI world. Thus, M&S teaching to develop skills in these areas remains highly relevant and valuable.

Similarly, an appreciation of the disciplines of mathematics and statistics is something that cannot be replaced by machines. The history of M&S is inextricably intertwined with human history, and to know and appreciate M&S history is to have an enriched understanding of human history, thought and development. In the words of Abd Algani (2022),

> The history of mathematics indicates that whenever a civilization placed a high value on mathematical ability, it made remarkable progress. Mathematics contributes to technology and scientific advancement.

Developing this appreciation in students remains valuable in ways that are unaffected by the advent of smart technologies.

A discrepancy exists between these educator priorities and students' own views as reported in the literature, which tend to emphasise computation as the primary purpose of M&S. In light of this, it may be beneficial for M&S educators to consider ways to reframe students' conceptualisations. When students perceive M&S as purely computational then this is what they will focus on, and the skills most relevant to an AI world will be the casualties.

7.3. Computational skills

Until the mid twentieth century, calculations were performed by humans, and computational skills necessarily played a pivotal role in M&S education. Some shift has occurred in M&S education since that time, perhaps more so in statistics than mathematics. The development of computing has substantially transformed the discipline of statistics. As discussed in the Introduction, given the computational demands of modern statistical practice, statistics education increasingly incorporates the use of computers for carrying out statistical computations. Most statistics educators we interviewed emphasised that the focus of statistics education should be on developing a higher-level understanding – knowing how to appropriately analyse any given data set – while treating the actual calculations, which are performed by software, as of secondary importance. This perspective was viewed as particularly important for upper-year students.

Mathematics lessons have also changed in some ways. Log tables are a thing of the past and pocket calculators are now ubiquitous. However, most of the mathematics educators interviewed maintained that mastering computational and procedural skills remains an important learning outcome for their students. Three rationales were put forward for this view. Firstly, students would need to be able to carry out these procedures in the future. Secondly, in service subjects, they were mandated by the partnering discipline. Thirdly, computational skills were viewed as a pathway to conceptual understanding. We wish to address each of these three points in turn.

A certain level of procedural fluency is essential for the practice of M&S. Mathematics can be viewed as a formal language, characterised by its own symbols, grammar, and thought structures (Silver, 2017). Engagement with M&S requires internalisation of these elements to the extent that routine reference to external sources is unnecessary, and this proficiency is developed through computational practice. Moreover, while a general appreciation may be acquired through lay texts, the epistemic integrity of mathematics rests on precision and rigour. It is not possible to develop deep conceptual understanding of a mathematical topic without the capacity to critically engage with and accurately interpret the formal structure and detailed logic of mathematical arguments.

Nevertheless, for humans, conceptual understanding is more valuable than computational proficiency. It underpins the creation of novel solutions to problems. Once deep conceptual understanding is developed, computational procedures can be efficiently executed using technological tools. As highlighted in the Introduction, computational skills devoid of conceptual insight offer minimal value in modern practice.

There is value in reframing unit outcomes to emphasise higher-order skills rather than computational techniques. For example, an outcome focusing on conceptual understanding – such as 'engineering students can set up matrices to solve simultaneous equations and interpret the resulting solution spaces' – may be more valuable than one stating 'engineering students can apply Gaussian elimination, cofactor methods, and Cramer's rule to solve simultaneous equations'.

If computational skills are taught primarily to meet external expectations, it is important to reconsider the intended learning outcomes. For instance, do electrical engineering

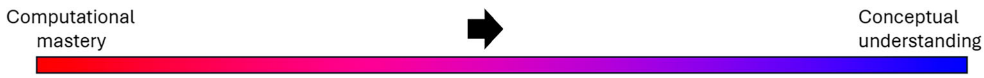

Figure 2. Students develop skills starting with computational mastery and working towards conceptual understanding.

students actually need to know Cramer's rule, or is the ultimate goal for them to analyse circuits effectively? It may be that the particular tools used – including technological tools – matter less than the development of strong analytical skills.

The argument that computational skills serve as a necessary pathway to conceptual understanding is examined in the following section.

7.4. Rethinking the relationship between computational mastery and conceptual understanding

A traditional approach to developing both computational mastery and conceptual understanding in M&S broadly aligns with Bloom's taxonomy. Instruction typically begins with declarative knowledge – for example, a theorem and its proof – followed by worked examples and practice problems that allow students to apply the theory in straightforward ways. Students are then presented with more advanced tasks that require novel applications of techniques, deeper theoretical analysis, and synthesis of ideas across domains. Through this progression, students gradually build both procedural fluency and conceptual insight. This staged development is illustrated in Figure 2.

However, M&S units are constrained by limited instructional time, and both computational and conceptual understanding require sustained effort to develop. This raises an ongoing pedagogical dilemma: how can educators balance the need for procedural mastery with the goal of fostering deep conceptual understanding? Where should this balance be struck?

Regardless of where the balance is set, this model presents several challenges. First, beginning with procedural instruction inevitably places early emphasis on computation, reinforcing the misconception that M&S is primarily about performing calculations. Second, many students do not progress beyond the procedural stage, further entrenching the belief that computation defines the discipline. Third, as highlighted in the educator interviews, this creates a genuine tension – especially for those teaching first-year courses – who often feel constrained in their ability to cultivate conceptual understanding due to the pressure to cover essential computational techniques. This tension was reported both by mathematics educators and by some statistics educators whose programmes followed this traditional teaching model. As noted, however, statistics educators are increasingly moving to a model that prioritises big-picture understanding, and leaves the procedural details to the computer, although students still need to be fluent in the procedures used to obtain appropriate results from the computer.

To address the persistent tension in M&S subjects where computational mastery remains essential, we propose a reframing of content delivery that treats M&S as an experimental science, analogous to physics or chemistry. Rather than teaching procedures as methods for solving specific classes of problems, procedural steps are introduced within the

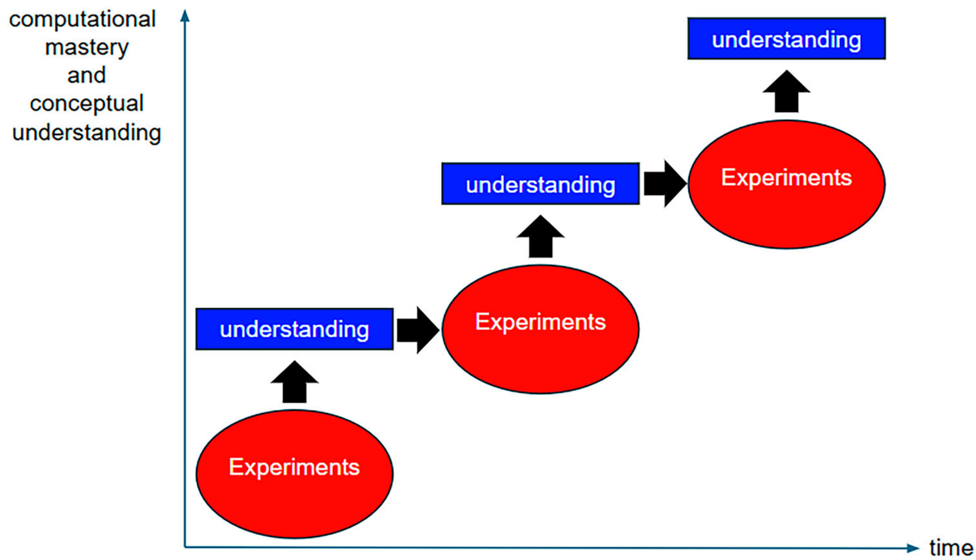

Figure 3. An alternative perspective in which M&S are viewed as experimental sciences, like physics and chemistry. The purpose of an experiment is to gain some conceptual understanding. Computational skills are taught as tools when needed to carry out the experiments, not as ends in themselves.

context of structured experiments designed to uncover conceptual understanding. Each new insight prompts further inquiry, leading to successive experiments and deeper understanding. In this, we embrace the iterative relationship between procedural knowledge and conceptual understanding described by Rittle-Johnson et al. (2001). This iterative process is illustrated in Figure 3. In this model, conceptual understanding and investigative questions sit at the centre of learning, while computational mastery is presented as a tool to support that end – much like measurement techniques in chemistry, which are necessary for experimentation but not the ultimate goal.

For example, a traditional lesson might begin by teaching students how to compute the determinant of a square matrix A, then explain its relationship to the solution set of $Ax = b$, followed by practice problems. In contrast, an experimental framing begins with a conceptual aim – such as investigating how the singularity of a matrix affects the solution set. The methodology introduces necessary tools, such as computing the determinant, as part of an investigation. The lesson mirrors the structure of a scientific report: aim, method, results, discussion, and conclusion. In this approach, students focus on designing appropriate methodologies, interpreting outcomes, and articulating insights – placing understanding, not calculation, at the centre.

This structure also supports the development of mathematical communication, as students justify and explain their findings, synthesise new insights with prior knowledge, and reflect on broader implications. The emphasis shifts from performing calculations to engaging in mathematical reasoning and inquiry. Crucially, this is not a move toward self-directed learning, but rather a reframing of the relationship between procedural skills and understanding – one that makes clear that procedural mastery supports conceptual development but is not an end in itself.

Viewing M&S units through an experimental lens shifts the emphasis from mastering a fixed set of procedural techniques to developing higher-order skills – mathematical thinking, communication, and appreciation – alongside a deepening understanding of the subject matter. This meaningful learning remains highly relevant in today's AI-rich environment. As students become more advanced and adept, they develop increasingly sophisticated mental models that enable them to operate at higher levels of abstraction, from which they can derive conceptual understanding directly from traditional expository theorem – proof discourse.

This approach also cultivates the dispositions and skills necessary for lifelong learning in M&S. As disciplinary knowledge continues to expand, M&S curricula risk becoming increasingly overloaded. Many educators we interviewed expressed concern that there is consistently more content to teach than students have time to meaningfully absorb. In an era where computers can perform many routine procedures, it is no longer essential for students to learn every algorithm by hand. Instead, it is arguably more valuable to foster curiosity, conceptual insight, and the ability to continue learning independently. Overcrowded content can lead to shallow learning and disengagement. There does need to be a balance between breadth and depth, particularly in the junior years of study, but prioritising depth – such as exchanging extended practice on hand-solving 3×3 matrices for time spent exploring relationships among solutions and vector spaces – offers students a stronger foundation for lifelong learning.

7.5. Study limitations

For this study we interviewed people who were active in teaching M&S and who had a strong interest in M&S education. We did not interview theoreticians who research teaching and learning theory in M&S but who do not teach students themselves. We also did not interview researchers who teach but do not have an interest in education. In doing so we have captured the views of M&S educators who had thought about these issues in the context of their own students, but they represent only a subset of the M&S community.

Only five of the interviewees were specialist statistics educators who taught exclusively statistics units, while the remaining ten who taught statistics also taught mathematics. Although we did not observe clear differences in perspectives between those teaching only mathematics and those teaching both mathematics and statistics, the small number of specialist statistics educators limits our ability to explore nuanced distinctions that might exist between the two groups.

In the interviews we asked educators to talk specifically about the units of study that they teach, rather than about their views of M&S as a whole. This may have had implications for the goals educators had for their students. For example, skills such as the ability to generalise, or the need to develop lifelong learners, were not raised by interviewees, whereas they may have been raised if the questions had been about the purposes of M&S education in general. On the other hand, if there are knowledge and skills that are deemed important in M&S and they are not being specifically addressed within units, it may be that students are not actually being taught these things and it is assumed that they will somehow absorb them by osmosis. Investigating to see if there are differences between general conceptions of what a M&S student should gain and specific unit goals would be an interesting piece of future research.

8. Conclusion

We opened this paper by raising questions about the impact of computing power and AI on M&S education. The issues are significant, and are perhaps best summed up in the words of one of the educators:

> I think we also are facing an existential crisis really because at the moment I struggle to explain to people why mathematics is going to be needed in the future. Because, I can imagine reasons that it will be very valuable in the future, but when somebody sees that computers can answer all the questions we teach an undergraduate they're going to wonder why we should teach the undergraduates this, and so I think we need to work very hard to actually rethink what it is we think is valuable about mathematics and redesign what we're teaching from all levels from primary school up to university. (Ed 20)

The work in this paper is an attempt to begin to do exactly this – to 'rethink what it is we think is valuable about mathematics [and statistics]' in order to inform the design of meaningful and valuable learning and teaching programmes.

Three questions were posed, and addressed. The first question asked what knowledge and skills were deemed most important by M&S educators. The next question asked in what ways the M&S educators' views aligned with the reality of an AI-rich world, and in what ways they were in conflict. The third question asked for ways in which M&S education could adapt current typical educational practices that were not well aligned with the needs of an AI-rich world.

In interviews we found four major themes describing what educators wanted students to gain from M&S studies. Three of these – mathematical thinking, mathematical communication, and mathematical appreciation – are all valuable in an AI-saturated world.

The fourth theme, content mastery, was divided into two sub-themes: computational mastery and conceptual understanding. A tension between these skills arises due to the time required to develop each and, because computational mastery precedes conceptual understanding in many courses, many students fail to reach adequate levels of conceptual understanding. Furthermore, as discussed in the literature, many students equate M&S with computational proficiency.

To address these challenges, we propose an alternative framework that views M&S units through the lens of experimental science. Learning is structured as a series of experiments, each aimed at developing specific conceptual insights, with computational skills introduced as tools to carry out the experiments rather than as ends in themselves. This reframing shifts the focus – and students' perceptions – from procedural skills toward higher-order understanding.

We hope that the findings and ideas presented here encourage further conversation on what it is that we, as mathematicians and statisticians, are trying to impart to our students and how changes to the way we teach could improve student outcomes and the general public perception of M&S.

Disclosure statement

No potential conflict of interest was reported by the authors.

Data availability statement

The participants of this study did not give written consent for their data to be shared publicly, so due to the sensitive nature of the research supporting data is not available.

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