UNCERTAINTY AND ERROR ANALYSIS

REVIEW: Significant Figures and Scientific Notation

Indicate how many significant figures there are in each of the following measured values.

1. 206.12 - 5
2. 300.750 - 6
3. 112.3 - 4
4. 0.198 - 3
5. 7.007 - 4
6. 0.00120 - 3

Convert each of the following into scientific notation.

1. 2,985 - 2.985×10^{3}
2. 0.00177 - 1.77×10^{-3}
3. 123,456 - 1.23456×10^{5}
4. 692 - 6.92×10^{2}
5. 0.000984 - 9.84×10^{-4}
6. 0.408 - 4.08×10^{-1}

Measurement always have some degree of uncertainty due to anavoidable errors.

1. **Error**

   -deviation of a measured value from the expected or true value.

2. **Uncertainty**

   -way of expressing this error.

The equation below shows the relationship of these factors.

measured value = (true value (±) uncertainty) units

ACCURACY VS. PRECISION

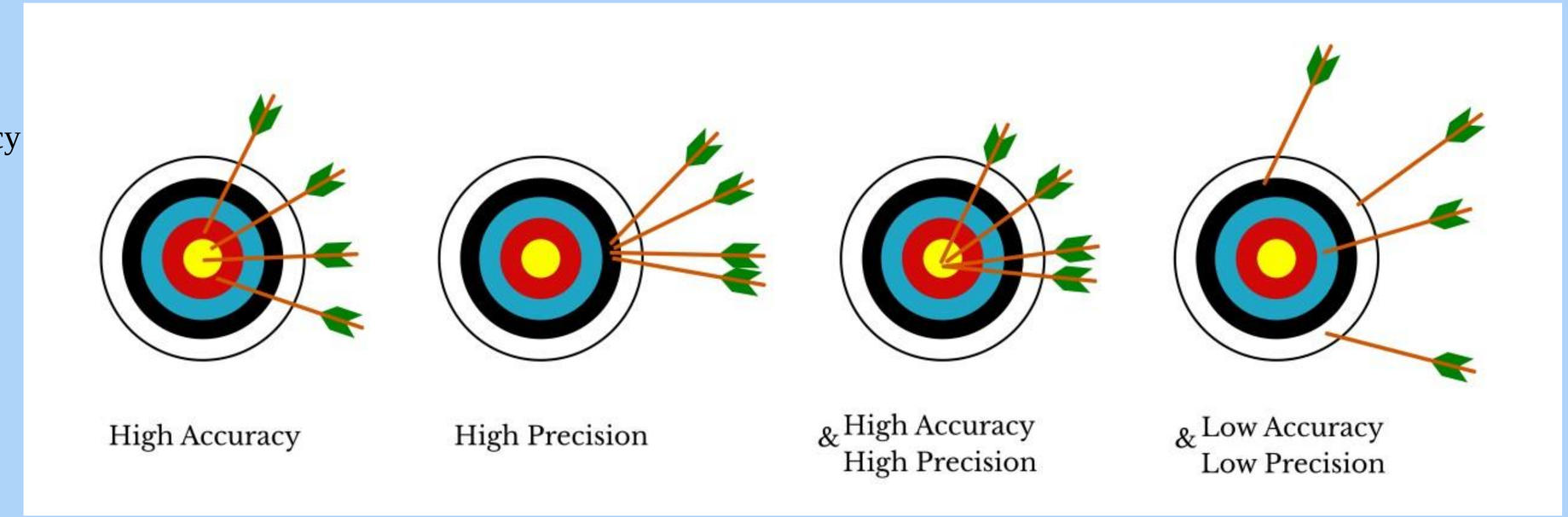
Accuracy

-refers to the closeness of a measured value to the expected or true value of a physical quantity

Precision

-represents how close or consistent the independent measurements of the same quantity are to one another.

Precision is how close the arrows are to each other , while the accuracy is how close an arrow is to the target. The red portion of the circle represents the expected result, while the arrows are the measurements.



Sample Problem 1. Consider the two illustrations below. How do these illustrations differ from each other? Which of the two illustrations represents accuracy and precision.

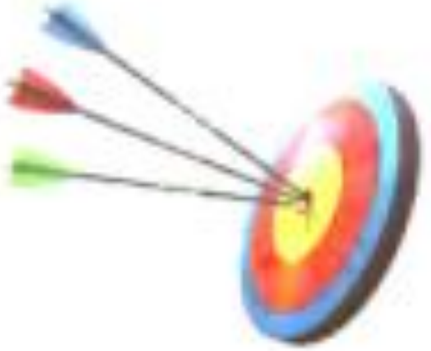


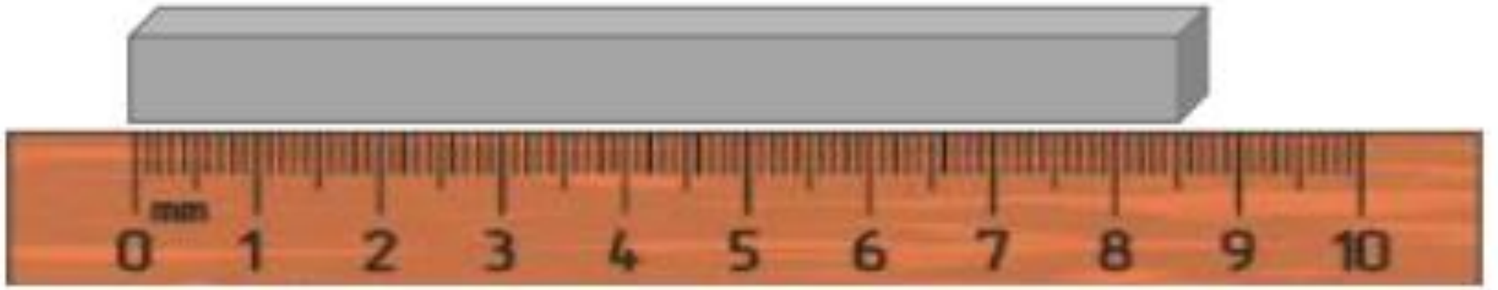
Illustration 1

Illustration 2

(*https://byjus.com/maths/precision/*
https://thetalkteam.com/the-power-of-intention/)

Illustration 1, shows the archer hits its target the bull's eye. Hitting indicates accuracy, while illustration 2 shows that the archer was not able to hit its target but its arrows are very close to each other, this represents precision.

Sample Problem 2. In the class discussion, Teacher X presents a block of wood and allows the students to measure its length. Four students are asked to measure and the data is provided below.



(https://www.pinterest.com.au/pin/720716746605520431/)

Participants	Length of the Wood
Student A	8.40 cm
Student B	8.49 cm
Student C	8.495cm
Student D	8.50 cm

From the given data, identify accuracy and precision.

Since accuracy defines as how close the measured value is to the true value, then it shows that student D gives a value of 8.50 cm as the length of the block of wood which is the true measurement of the block of wood. On the other hand, the measurements given by other participants are repeated and they are very close to each other thus, those measurements are considered precise.

RANDOM VS. SYSTEMATIC ERRORS

RANDOM ERRORS

-result from unpredictable or inevitable changes during the data measurement.
-affect the precision of the measurements
-may be reduced by increasing the number of trials of a measurement and averaging the results.

SYSTEMATIC ERRORS

-usually come from the measuring instrument or in the design of the experiment itself
-these errors limit the accuracy of one's results.

PERCENT ERROR AND PERCENT DIFFERENCE

PERCENT ERROR

-calculated when there is an expected or true value of a quantity
-compares an exact value to an approximate value

$$\text{Percentage Error} = \frac{|\text{Measured Value} - \text{True Value}|}{|\text{True Value}|} \times 100$$

$$\text{Percent error} = \frac{|x - x_T|}{x_T} \text{ x } 100$$

PERCENT DIFFERENCE

-measure of how far apart the different measured values are from each other
-compares two average values

$$\%\ \text{difference} = \frac{|\text{value 1} - \text{value 2}|}{\left(\frac{\text{value 1} + \text{value 2}}{2}\right)} \times 100\%.$$

$$\text{Percent difference} = \frac{|x_1 - x_2|}{\frac{x_1 + x_2}{2}} \text{ x } 100$$

SAMPLE PROBLEM 1

Two trials were performed in an experiment to determine the latent heat of vaporization (Lv) of water at 100°C. The values of Lv of water obtained were 532 cal/g and 536 cal/g.

a.) Find the percent difference between the two values.

b.) Find the percent error for each measurement if the accepted value pf Lv of water at 100°C is 540 cal/g

VARIANCE

-measures the squared deviation of each number in the set from the mean.
-another way to test errors from multiple measurements of a physical quantity

A variance of zero (0)
- means all measurements are identical

A small variance
- indicates that the values are close to one another, which means they are precise

The variance of a set of measurements is calculated step-by-step as follows:

1. Take the mean of the set of measurements, $\bar{x} = \frac{\sum x}{N}$
2. Take the deviation of each measurement from the mean $(x - \bar{x})$
3. Square each deviation, $(x - \bar{x})^2$
4. Get the sum of the squares of each deviation, $\sum(x - \bar{x})^2$
5. Divide the sum of the squares by the number of measurements in the set, $\frac{\sum(x - \bar{x})^2}{N}$

   In symbols, variance (σ^2) is

$$\sigma^2 = \frac{\sum(x - \bar{x})^2}{N}$$

STANDARD DEVIATION

-measure of how diverse or spread out are a set of measurements from their average.

$$\sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{N}}$$

Small standard deviation

-means that most of the measurements aare close to their average

Large standard deviation

-means that the measurements are very diverse

The measurement of x of a physical quantity in a set of measurements is usually reported as

$$x = \bar{x} \pm \sigma$$

where $\bar{x}$ is the mean of the set of measurements and σ is the standard deviation of the measurement

SAMPLE PROBLEM

During an experiment in a physics laboratory class, a group of five students was asked to measure the period of a simple pendulum. Their measurements were as follows: 2.3 s, 2.4 s, 2.2 s, 2.5 s, and 2,1 s. Determine the

a. mean
b. variance
c. standard deviation
d. measured period of the pendulum

HAVE A GO YOURSELF!

In an experiment, 10 trials were done to determine the range of a projectile. The measurements for the range of the projectile in centimetersb are as follows:

134.8	133.9	135.1	134.7	135.3
134.9	135.2	134.8	135.5	135.4

Determine the (a) mean, (b) variance, (c) standard deviation, (d) measured range of the projectile