UNIT - 1

Introduction to Primary School Mathematics

Notes

1

NATURE OF MATHEMATICS

1.1 INTRODUCTION

Mathematics is an inherent part of human thinking. It has emerged from humanity's need for socialisation and development, and has matured into a discipline of abstract yet useful knowledge. Many civilisations have contributed significantly to the development of mathematics. During the 19th century, classical mathematics, which was based on highly standardised protocols, was challenged by more flexible approaches to doing and applying mathematics. The era of modern mathematics allowed scientists, social scientists, linguists, and others to apply classical mathematical theories and principles to new explorations. Present-day mathematics is a constantly growing field, surpassing existing boundaries of knowledge and innovation. The foundation for such an exciting approach to mathematics learning starts in the early years. Therefore, it is essential for mathematics teachers to have a sound understanding of the nature of mathematics, which should be reflected in the primary mathematics curriculum.

In the sections below, we will discuss mathematical concepts, their structure, and their mathematical language. Practicing teachers are encouraged to read and engage with the content by reflecting on their classroom experiences.

1.2 LEARNING OUTCOM ES

After completion of the chapter, the learner:

- develops insights into the nature of mathematics as a discipline and its constructs;
- discusses and illustrates the nature of mathematical concepts, their properties, and interrelated hierarchical conceptual structures;
- visualises and interprets everyday experiences through mathematical understanding;
- explains and uses the structure of mathematical language, including symbols, notations, and vocabulary, with primary learners;
- appreciates and correctly uses everyday language in mathematics learning with young learners; and
- develops pedagogical strategies to integrate the nature of mathematical concepts, processes, structures, and language, in order to build strong mathematical foundations and a positive disposition in primary-grade learners.

1.3 NATURE OF MATHEMATICS: CONCEPTS AND CONCEPTUAL STRUCTURES

The word 'Mathematics' is explained differently by different users. Galileo believed mathematics is the language of the universe, while Einstein argued that the systems of the universe can be reduced to a mathematical equation. People use mathematics as a standard tool for day-to-day calculations and measurement, whereas a mathematician enjoys working with highly intricate and abstract structures of mathematical concepts. Whether an engineer uses the power of mathematics to design a flyover or a building, a weaver woman uses mathematical patterns in weaving a basket, or a fruit vendor applies compact mathematical design to optimize fruit arrangements, we all use mathematics

Notes

in multiple ways. As a dynamic human endeavour, mathematics is not a fixed body of knowledge and facts; rather, it has evolved through constant human effort to understand the world. Mathematics must be seen as an outcome of human curiosity, innovation, and problem-solving.

Mathematics is a complex network of interrelated concepts. The ideas that the human mind constructs through logic and abstraction are called mathematical concepts. This network grows both horizontally and vertically.

Let's start by exploring what encompasses the disciplinary nature of mathematics and the logical processes involved in it:

Mathematics is axiomatic in nature.

All mathematical concepts, procedures, and results are based on primitive statements called axioms, which we assume to be true. A set of axioms forms the foundation upon which carefully deduced mathematical results are built.

Mathematics deals with generalisation.

The beauty of mathematics lies in generalizing a mathematical concept from one case to multiple similar cases under exactly the same conditions.

Mathematics is both inductive and deductive.

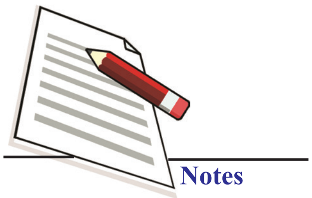
Most mathematical ideas grow through inductive logic, progressing from concrete to abstract and from particular to general. These, in turn, lead to carefully drawn mathematical results through strong deductive reasoning.

Mathematics has its roots in culture and time.

Though modern mathematics is highly abstract, with limited representation in the day-to-day functioning of many people, we must not lose sight of how mathematics has grown, developed, and travelled through civilisations such as the Egyptians, Indians, Babylonians, and others, and across continents, before evolving into a dynamic intellectual enterprise.

Mathematics is a dynamic human creation.

Although mathematics is considered one of the most exact sciences, it has undergone its share of limitations and corrections. The evolution of the present numeration system took centuries of human effort to become the most preferred number system. Mathematical paradoxes (Russell's paradox, Zeno's paradox), and the need to create alternate axiom systems (such as non-Euclidean geometry),



are examples that show how mathematicians have struggled to balance the traditional values of mathematics with the expansion of mathematical ideas.

Mathematics has its own language.

Formal mathematics is a disciplinary field with its own language. Like any language, it has its own vocabulary, structure, notation, and symbols. This language is precise, sophisticated, and symbol-based. Proficiency in mathematical language is necessary to learn, use, and appreciate mathematical knowledge. (You will learn more about mathematical language in a later section of this chapter.)

Mathematics is a creative endeavour.

Mathematics provides multiple opportunities to represent and work with mathematical ideas in different ways. Many such opportunities arise from creative applications of mathematical knowledge in interdisciplinary fields.

Here is a pictorial representation depicting the interconnection of different forms and sources of mathematical knowledge:

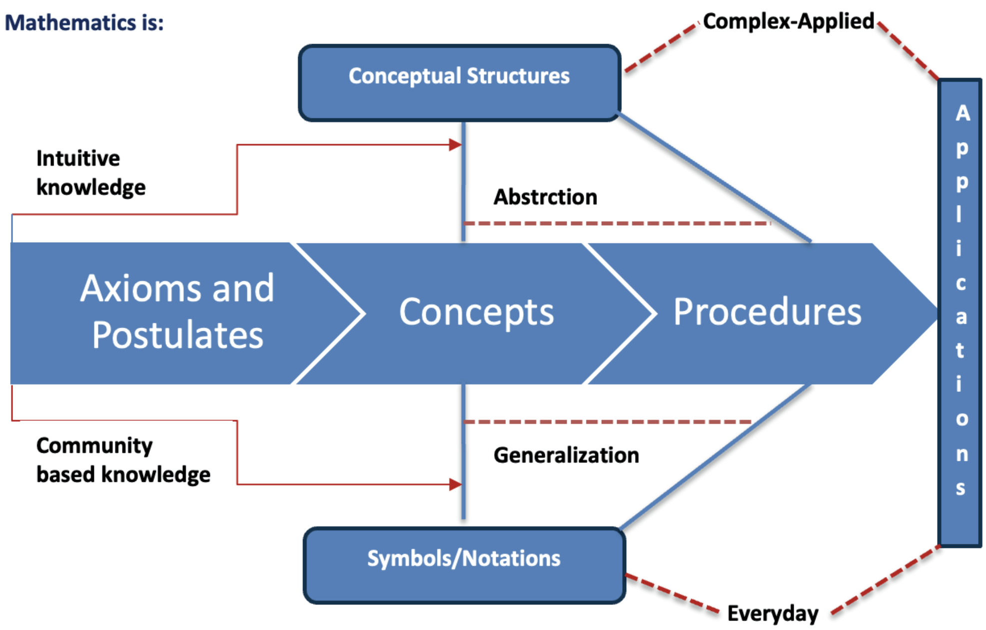


Fig 1.1

A strong foundation in basic mathematical concepts is crucial for young learners. Early skills such as counting, understanding numerals, and recognizing shapes and measurement serve as the building blocks for more advanced mathematical skills needed in the future. Mathematics learning experiences in the primary years shape learners' self-beliefs and attitudes towards the subject. This journey begins with the mathematisation of everyday experiences.

ACTIVITY 1.1

Ask learners to:

(a) Identify and make a list of patterns they observe around them, such as:
- The pattern of sunrise and sunset
- Patterns in a calendar
- Patterns of shapes that tessellate

(b) Represent these patterns through drawings or using any concrete materials.

(c) Describe each pattern mathematically.

(d) Present and explain the reasons and rules that define each pattern.

CHECK YOUR PROGRESS 1.1

Mathematics is considered an abstract subject, however, it should be taught in concrete ways in preparatory years. Do you agree with the statement? Justify your claim reflecting on your classroom experiences.

1.4 MATHEMATISATION

Mathematisation is our ability to describe and understand our experiences through a mathematical lens. This process occurs when a learner converts a real-world context into a mathematical representation, using mathematical symbols and vocabulary. We can consider mathematisation at two levels: everyday mathematisation and advanced mathematisation.

Everyday mathematisation takes place when a child uses basic mathematical concepts—such as measurement, quantification, grouping, sequencing, and others—in everyday situations or early math learning. For example, when sharing a chocolate bar with two friends, the child applies the concept of fractions and uses appropriate vocabulary by sharing 1/3 of the chocolate with each friend.

Everyday mathematisation allows children to:

- visualise mathematical concepts around them;
- solve simple, day-to-day problems using basic mathematical concepts;
- use mathematical language to make sound judgments in everyday situations.

Everyday mathematisation should be frequently reinforced by using mathematical concepts and language in daily activities and communication.

For example, instead of making a casual statement like, "Today is hotter than yesterday," the teacher can say, "Yesterday the temperature was 38°C, but today it is 40°C. That's why we are feeling more heat today." It requires a conscious effort on the teacher's part to encourage children to use their mathematical learning beyond mathematics textbooks.

Abstract mathematisation involves translating abstract elements into a mathematics problem. This is a vital competency because it helps students in higher classes learn how to use the best approaches to associate different concepts, expressions, and relationships in order to solve actionable mathematical problems (Cobb et al., 2021). This process is complex and generally involves first identifying the problem in context, collecting the necessary information, assigning and determining variables, and developing a resolution model (Duyen & Loc, 2022). Further stages include deriving a potential solution and interpreting the outcomes. Polya described this process as problem solving, while mathematisation as a skill lays the foundation for mathematical modelling, a process widely used in applied mathematics across countless applications.

1.4.1 Mathematical Thinking and Learning

Mathematical thinking begins with identifying patterns through logic and abstraction, and then building more complex patterns based on these foundational ones. Even the simplest mathematical activity, such as counting, involves abstract thinking and logic.

Here is an example highlighting the process of counting balls:

Teacher: Count the yellow balls.

Child: Three yellow balls

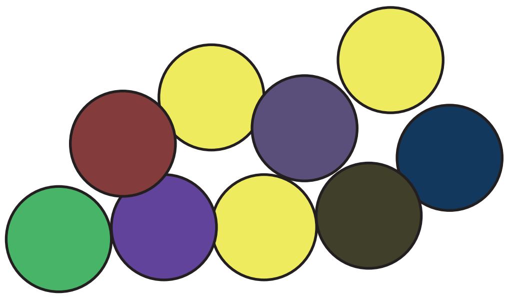


Fig 1.2

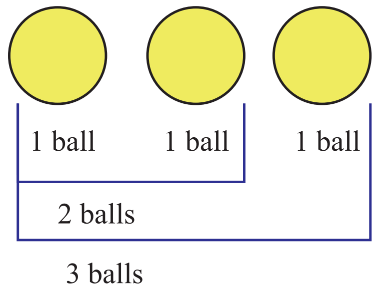


Fig 1.3

The concept *three* is based on the following two processes:

- counting each ball as a single ball (or one unit);
- putting them as a collection, based on the principle of similarity *(as balls are similar in color and size).*

Mathematics is more than just working with numbers, performing operations, or solving routine problems. Mathematical thinking encompasses logical reasoning, abstract thought, and problem-solving, enabling us to make sense of the world through numbers, patterns, measurement, and quantification. Logical reasoning involves observing patterns and relationships based on certain rules and drawing conclusions. Abstract thinking in mathematics is the ability to move beyond concrete examples, generalise patterns, and represent them using symbols.

1.4.2 Conceptual and Procedural Knowledge

Mathematics consists of both conceptual structures and procedural systems. Effective mathematics learning requires an understanding of both concepts and the procedures associated with them. Conceptual knowledge refers to understanding mathematical concepts and their underlying relationships and principles. Procedural knowledge, on the other hand, is the ability to carry out specific procedures or algorithms to apply those concepts. It includes proficiency in using standard methods and operations.

For example:

- Understanding multiplication as a special case of repeated addition demonstrates conceptual understanding.
- Multiplying 13 by 24 using a multiplication algorithm demonstrates procedural knowledge.

In classroom learning, conceptual knowledge should precede procedural knowledge. Primary grade learners should be given plenty of examples, non-examples, and contextual variations using concrete manipulatives, games, and real-life activities. This approach helps them learn to abstract a concept independent of its physical embodiment.

1.4.2.1 Mathematical Abstraction

Mathematics can be considered from two primary dimensions. On one hand, it can be described through concrete concepts and principles (Quigley, 2021).

Notes

In this case, children learn the underlying principles that govern mathematical operations. Mathematics involves a transitional journey: from concrete experiences to pictorial representations, and then from pictorial to abstract notations, which are used for complex computations and analyses in higher classes. In the early stages, concrete and pictorial experiences enable students to relate to mathematical problems using objects and real-world contexts (Bruner, Dienes, Quigley, 2021). We can recall Bruner's enactive-iconic-symbolic model, Dienes' Dynamic Principle of Learning, and subsequent research emphasizing the importance of spending considerable time helping children internalise concepts through concrete experiences.

For example, during the primary years, mathematics learning often encourages the use of blocks or real-world objects to help learners master basic computations. In the pictorial stage, instead of using blocks, learners may draw small circles or other shapes to help them add or subtract according to the given problems. The final stage involves the use of symbols to solve more complex mathematical problems. At this level, learners can comprehend and translate word problems into mathematical equations to develop solutions. It is important to encourage primary grade learners to form equations using concrete objects, pictorial representations, and eventually mathematical symbols. This progression helps them understand how mathematical solutions are derived from equations.

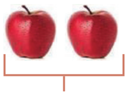
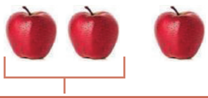
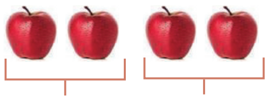
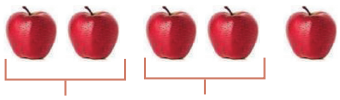
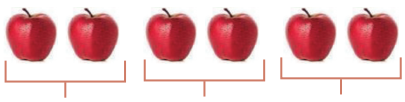
1.4.2.2 Mathematical Generalisation

In mathematics, concepts and processes are often developed by identifying and describing patterns based on predefined or existing properties (Hunter & Miller, 2022). This makes it easier to apply mathematical elements across a wide range of contexts, which helps in problem-solving. The primary advantage of this approach is that it enables learners to generate patterns that foster understanding of relationships between numbers. Simple observations can lead to the creation of general rules. In the primary years, teachers can begin with simple examples and gradually move towards more complex and scientific generalisation of mathematical results. For example, teachers might help learners discover through examples that the order of numbers does not affect the result in addition—then extend this idea to fractions, and later to more complex cases such as the commutativity of addition for different sets of numbers, matrices, or angles.

The ability to generalise is essential for learning mathematical proofs and understanding the difference between proof and verification.

For example, in counting the sequence of numbers, instead of introducing even and odd numbers, children can be encouraged to find the patterns by breaking down numbers in the groups of twos, and forming a hypothesis about numbers that can be paired in twos without any remainder.

Table 1.1

Number	Objects	Observation
1 apple		Can't be paired in two
2 apples		One pair of two apples
3 apples		One pair of two apples and one remaining
4 apples		Two groups of two apples and zero remaining
5 apples		Two pairs of two apples and one remaining
6 apples		Three pair of two apples and zero remaining

Encourage children to frame hypotheses:

Hypothesis-1: If we break down a number in a group of twos, the remainder can be zero or one

Hypothesis-2: If we break down a number is a group of twos, the remainder will always be zero or one.

Now the teacher can introduce that the numbers with zero remainder are called even numbers and numbers with one remainder are called odd numbers.

The principle of generalisation is an essential feature of the disciplinary structure of mathematics.

ACTIVITY 1.2

Ask learners to:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Circle all the numbers divisible by 2.
- Cross all the numbers multiple of 2.
- Write your observations and frame a hypothesis.
- Verify the hypothesis for numbers between 100-200.
- What can you say about the relation between multiplication and division operators?
- Discuss the nature of placement of even numbers on the grid.
- Make a sequence using even numbers with smallest even number as the first term and difference between any two consecutive terms shall be 10. Colour the terms of sequence on the observation chart. Write an observation report.
- What fraction of numbers on the grid are even numbers?

CHECK YOUR PROGRESS 1.2

1. *Mathematisation* as an ability refers to:

   A. Mastering solving problems in a limited time.

   B. Converting real-life problem situations into mathematical problems and solving them with mathematical reasoning.

C. Practicing mathematical algorithms until they are automatic.

D. Communicating mathematical definitions and symbols efficiently.

2. A teacher asks students to observe the following numerical equations, and state the rule for adding odd and even numbers.

4 + 6 =10

7 + 3=10

3 + 6 = 9

8 + 2 =10

9 +1 = 10

4 + 5= 9

3 + 6 = 9

This activity helps in:

A. Practicing of number facts

B. Strengthening mental calculations

C. Mathematical generalisation through patterns

D. Mastering addition

Answers: 1. B 2. C

1.5 MATHEMATICS LANGUAGE, STRUCTURE AND NOTATION

Mathematics can be considered a language with its own specific syntax. The language of mathematics includes symbols and notations that represent concepts, operations, units, and quantities. It also makes use of notations such as parentheses, fractions, decimals, exponents, and symbols for differentiation and integration, among others. These symbols and notations are precisely defined, each with a clear and unambiguous meaning.

For example, the mathematical expression 3/4 represents 3 parts out of 4 equal parts of a whole. The nature of this quantity can vary—it could be an apple, a piece of ribbon, or even a bunch of bananas. The 3/4 representation is precise, abstract, and generalised, conveying the same exact meaning regardless of the context. This level of precision ensures that mathematical arguments remain logical and verifiable. In classroom settings, emphasizing precise definitions—

such as those for prime numbers or irrational numbers—helps students avoid misconceptions and build a strong conceptual foundation.

The language of mathematics allows mathematicians to express abstract and complex mathematical relationships clearly. For instance, symbols such as "+" and "–" are used for addition and subtraction (Erath et al., 2021), and may also denote positive and negative values. The symbols "×" and "÷" represent multiplication and division, while "=" signifies equality. The symbols "<" and ">" are used for logical comparisons.

Mathematical symbols and notations connect conceptual and procedural knowledge for practical applications. Mathematical concepts are represented using symbols, which are then applied in procedural algorithms. The interdependency between mathematical concepts, procedures, and language is illustrated below:

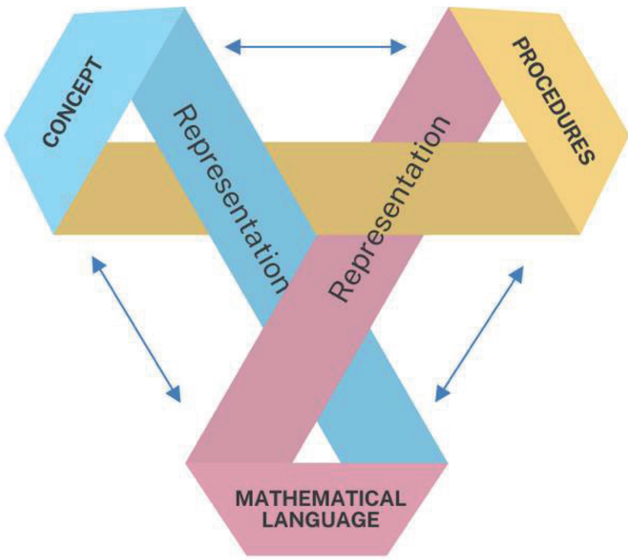


Fig 1.4 Rotation between Mathematical Concepts, Procedures and Language

Riccomini et al. (2015) describe comfort with mathematical language and communication as a necessary prerequisite for understanding and applying mathematics. They further highlight that effective use of mathematical language involves a strong mathematical vocabulary, fluency with numerals and symbols, and the ability to correctly comprehend and use them. For example, using different mathematical terms, such as, *find the total, add the numbers, how many in total*, represents the same operation of adding two or more quantities of the same units. Similarly, *one half of an apple, half of an apple, two quarter of an apple*, communicate the same meaning. Teachers should encourage frequent and fluent use of mathematical language in classroom discourse. A lack of mathematical language skills can hinder students from working fluently and accurately in mathematics.

Mathematical communication skills are developed through ongoing discussions and interactions between children and their teacher. These skills include learning how to create mental images of problems and visualise strategies to solve them. Such competencies are essential for mathematical thinking and effective problem solving (Erath et al., 2021). Fluency in mathematical language should be cultivated through frequent and varied forms of communication. Teachers should encourage children to speak mathematically, using appropriate vocabulary and symbols. Creating mathematical discussion forums in the classroom can further support this development. Identifying, associating, and applying mathematical symbols should be reinforced through both imagery and concrete activities. It is important for children to become comfortable, confident, and fluent in using mathematical language.

Notes

ACTIVITY 1.3

Ask learners to:

For a week, carefully record all the mathematical words/vocabulary they have used outside the mathematics classroom.

Make a dictionary in alphabetic order of all these words under following heads:

Word	Meaning	Mathematical Symbol	Draw/Image/Example

CHECK YOUR PROGRESS 1.3

1. The most effective sequence of introducing the fraction in preparatory grades is:

   A. Symbolic representation → Conceptual understanding→ Visual form

   B. Visual form → Conceptual understanding → Symbolic representation

   C. Conceptual understanding →Visual form → Symbolic representation

2. Reflect on teaching experience to explain why it is important for young learners to develop correct understanding and usage of mathematical language.

Answers: 1. C

1.6 MATHEMATICS IN EVERYDAY LIFE

Mathematics is a universal language and subject that is used in daily life experiences. People apply mathematics in everyday situations—often knowingly, but sometimes unknowingly—to solve problems and make decisions (Duyen & Loc, 2022). Three of the most common areas that require a strong mathematical foundation are measurement and quantification, understanding time, and managing money.

For example, timekeeping requires basic calculations to understand how minutes, hours, days, and weeks are organised. This knowledge can also be applied to everyday activities, such as cooking, where math is used to determine the best combinations and quantities of ingredients. In large-scale cooking, it becomes essential to estimate the amount of food needed to serve a specific number of people, relying on fundamental mathematical concepts.

Additionally, mathematics plays an important role in entertainment. Many games require keeping score—volleyball, for instance, involves tallying the points each side earns over a given period. This type of computation forms the basis of common mathematical principles.

Budgeting and resource allocation at home also rely on mathematics. Setting daily, weekly, or monthly budgets involves the use of addition and subtraction to manage household expenses. Mathematical skills are equally important in data analysis, decision making, and logical or critical thinking. These competencies help people understand and interact effectively with the world around them.

Children should develop sufficient mathematical skills and competencies to perform everyday tasks confidently and accurately. The successful completion of such tasks often depends on the ability to allocate enough time to acquire the necessary mathematical competencies. For example, mathematics helps individuals calculate the total time required to complete an activity, while money management requires an understanding of basic math principles.

These everyday contexts also demand an understanding and use of different units of measurement, unit conversions, and a solid grasp of foundational computation, including place value, to interpret the magnitude of various measurements. Teachers can encourage children to identify and discuss activities in their daily lives where they use mathematics.

ACTIVITY 1.4

Create a discussion forum in the class where students will share how they use mathematics in day-to-day activities (for example, time management) and in more advanced uses (for example, comparing temperatures). Classify these activities as 'basic activities' that don't require much conscious use of mathematical knowledge and 'advanced activities' that require more conscious effort to think and use mathematical knowledge. Engage students in discussing how frequently, and in what different ways, we use mathematics.

CHECK YOUR PROGRESS 1.4

Choose a common real-life context from children's local context (e.g., visiting the vegetable market, planning an ice cream party, or baking a cake, or arranging classroom furniture).

Design small group activities for your class that can scaffold math learning.

1.7 MATH LEARNING IN PREPARATORY YEARS

In the early stages, children are introduced to mathematics through pre-number concepts such as classification, sequencing, and class inclusion, which help them make sense of numbers. Over time, they are introduced to counting, representing numbers on a number line, and the more advanced concept of place value, which provides an awareness of the magnitude and significance of each number. This progression also introduces children to a more formal and structured understanding of mathematics. Learning mathematics at this stage plays a vital role in shaping learners' attitudes toward the subject. A child who develops misconceptions or loses interest at this stage is likely to maintain the same attitude in higher classes. Teachers play a critical role in shaping children's attitudes toward mathematics.

ACTIVITY 1.5

Hands-on Activity:

Refer to Chapter I, Mathematics Mela, Class III, NCERT:

Activity: Lets' Do (page 3-4), to *count the number of letters in a name,* can be contextualised by asking children to count the number of letters in each other's names and extend the activity as follows:

- arrange them in ascending order
- arrange the sitting plan with same length of names
- make groups of names with same length
- make tally-bar table with the length of names
- represent information in pictorial form.

Length of names	Number of names

Such activities can reinforce the idea that there are multiple ways to use and represent mathematical ideas.

1.7.1 The Role of Teachers in Shaping Mathematical Minds

Teachers play a decisive role in shaping students' minds and attitudes toward mathematics. Many evidence-based theories suggest strategies for achieving effective and positive outcomes in mathematics learning (Bruner, 1960; Dienes, 1998; Turner, 2013). Within the classroom setting, teachers can refer to the CPA (Concrete-Pictorial-Abstract) framework, in which children are taught through a stepwise strategy until they master fundamental concepts and competencies. The CPA approach is based on Bruner's cognitive development theory of 'enactive-iconic-symbolic' stages of learning. This theory suggests that students should first master abstract mathematical concepts through concrete resources, then through pictorial representations, and finally by associating with abstract mathematical symbols. Mastery of mathematical concepts also involves understanding contextual variations, achieving fluency in application, and making meaningful connections.

When a teacher uses simple and easy-to-understand principles, children learn to make meaningful connections between abstract mathematical concepts and everyday experiences. This approach helps develop a positive mindset toward mathematics. Teachers can create an ecosystem that bridges the divide between formal school mathematics and the local mathematical knowledge children use in their daily lives. By making the math classroom lively and experiential, teachers promote mathematical thinking and decision-making.

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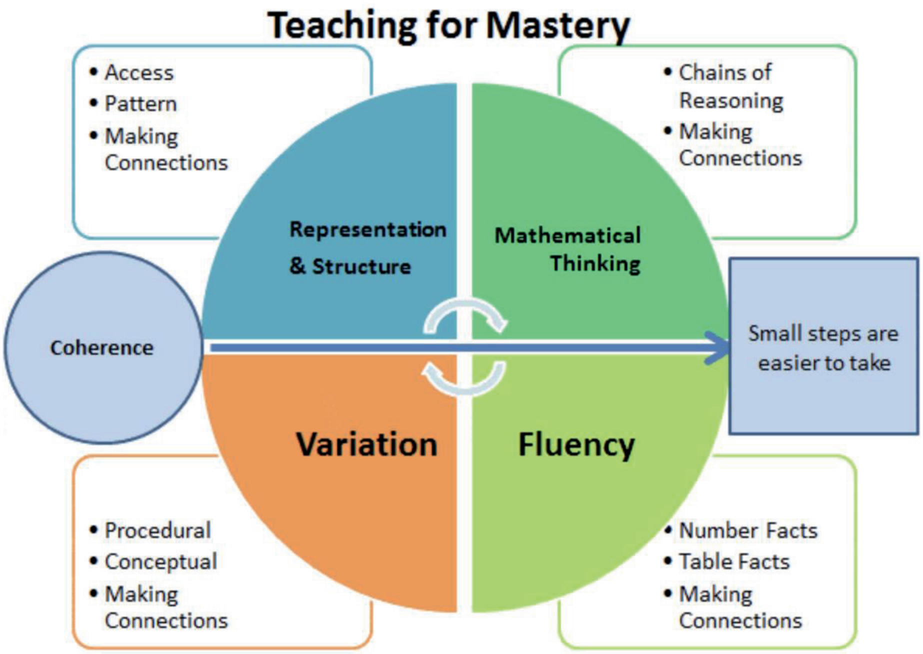

Fig 1.5 The Five Big Ideas in Teaching for Mastery (NCETM 2024)

When children are encouraged to bring their local experiences into the mathematics classroom, they feel more valued and responsible. Early mathematics learning experiences are significant for laying a strong foundation for advanced mathematics learning in later years. These experiences also help students gain fluency in mathematics through progressive exposure to practical situations. For instance, when teaching the fundamental operations of arithmetic, teachers should ensure that children have a solid understanding of numbers, numerals, place value, and how to use various operators. Fluency can be achieved by ensuring a deep conceptual understanding of problems. Allowing children to work with different problems involving various operators and combinations of operators helps them understand and appreciate mathematical properties such as commutativity and associativity. This understanding can be further strengthened through progressive problem-solving using math games, simple projects, and visual support resources.

In the next chapters, we will explore pedagogical and assessment strategies, and learn how to develop creative and engaging teaching and learning resources.

ACTIVITY 1.6

Create a CPA plan for *Introducing Zero as a concept and its place in the number system.*

CHECK YOUR PROGRESS 1.5

Ms. Seema, a fifth class math teacher is struggling to make her class engaging and inclusive. Suggest her a few strategies to:

- involve all learners, particularly girl students;
- make math learning more relatable to learners' experiences;
- promoting peer learning;
- encouraging questioning in the class.

1.8 LET US SUM UP

Mathematics teachers play a vital role in developing strong mathematical foundations and the right attitude among young learners. It is equally important for teachers to have a sound understanding of the nature of mathematics and the basic structure of mathematical knowledge. This chapter presents an overview of the essential components that primary mathematics teachers should be equipped with:

- Nature of mathematics and the structure of mathematical knowledge
- Mathematisation and mathematical thinking
- Language of mathematics
- Mathematics in real life
- The role of teachers in the mathematics classroom

1.9 END EXERCISES

1. Justify or refute the following statement with two examples:

   Mathematics is a fixed body of concepts and a set of procedures.

2. Seema is a class III mathematics teacher. She struggles to relate classroom learning to students' local experiences. Suggest two strategies to make classroom learning more relatable for students.

3. What are the prerequisites for the following?

   - Adding two quantities
   - Subtracting two fractions
   - Introducing zero

4. How would you help learners differentiate between common words used in everyday language and those used in mathematical vocabulary?
5. Identify common mathematical misconceptions among primary grade learners. What are the pedagogical challenges in rectifying these misconceptions?
6. Explain how you would introduce the concept of place value or fractions using concrete materials in an inclusive classroom.

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