DYNAMICS OF SINGLE-AXIS DRIVE SYSTEMS

DEPARTMENT OF
ELECTRO-MECHANICAL TECHNOLOGY

Engr. Edwin C. Cuizon, Jr.

The word dynamics means how things move when forces or torques act on them.

Joint Axis Drive System

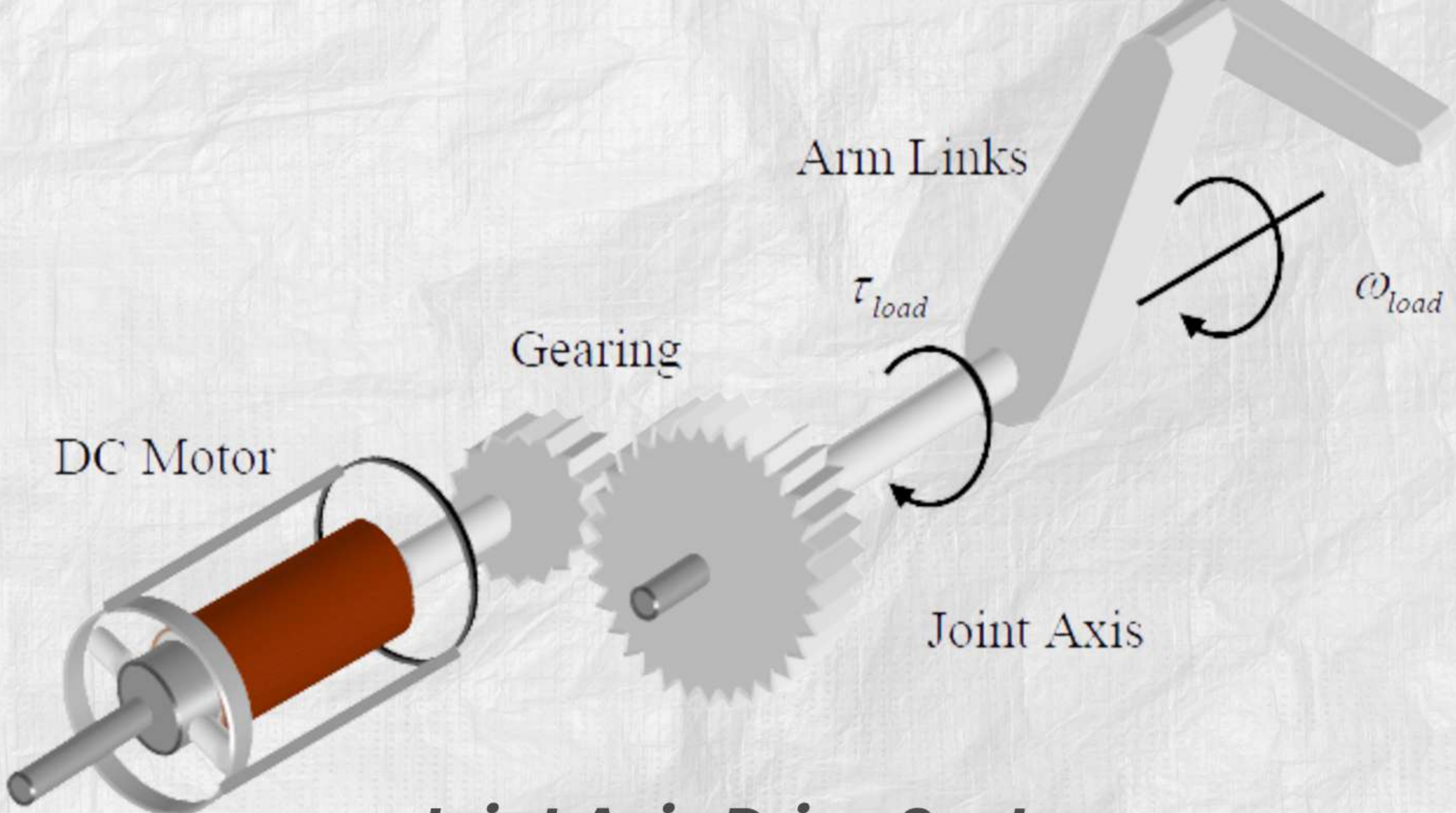
SINGLE-AXIS DRIVE SYSTEMS

A single-axis drive system is a mechanism that controls motion along one rotational axis.

SINGLE-AXIS DRIVE SYSTEMS

The basic structure is:
Motor → Gear Reducer → Load

The motor provides motion, the gear reducer modifies torque and speed, and the load is the component being moved.

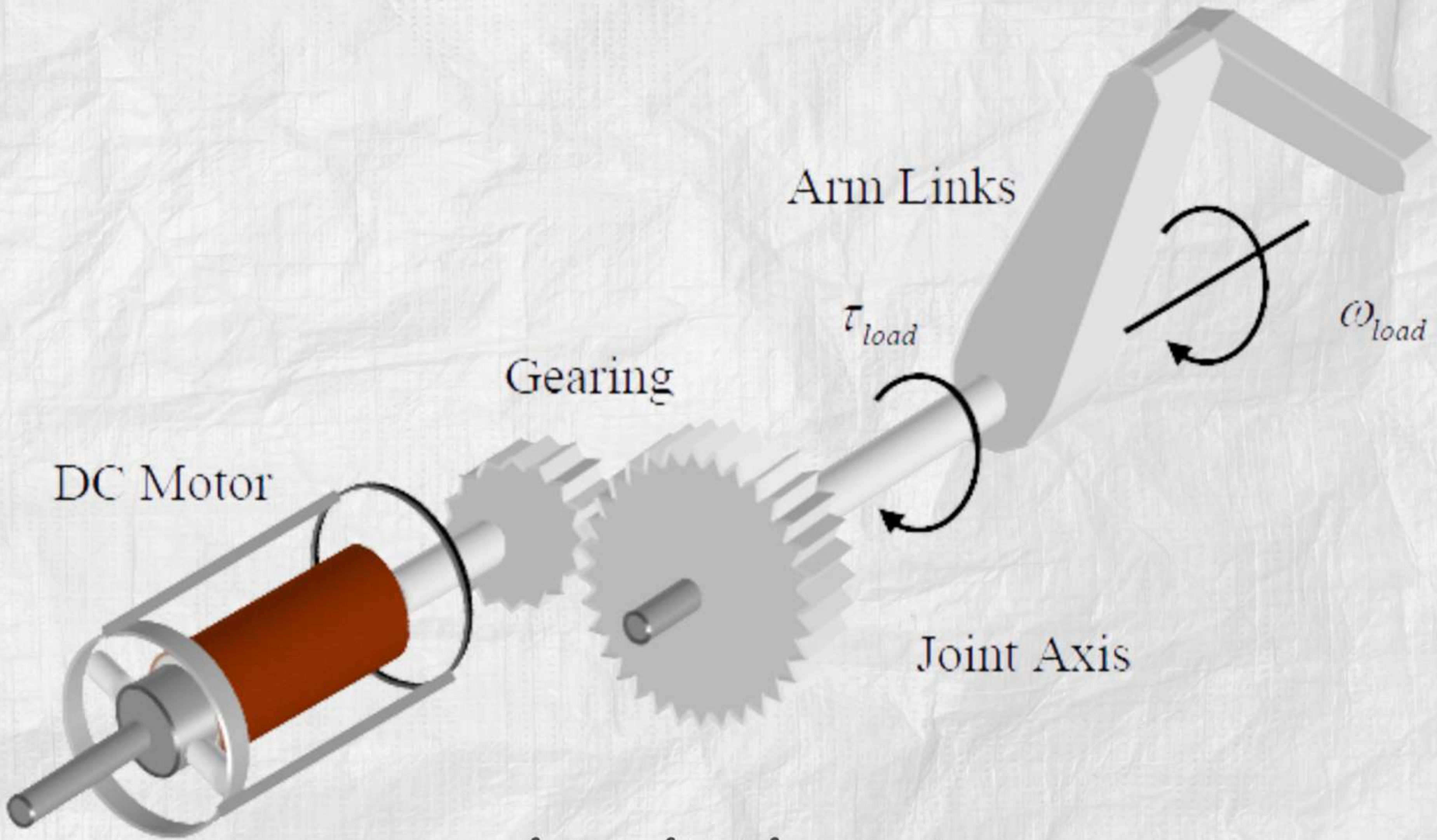


Joint Axis Drive System

Why do we need a Gear Reducer?

A DC motor naturally operates at high speed and low torque. However, robotic joints usually require low speed and high torque

The motor is connected to a gearbox. The gearbox acts like a multiplier. It reduces speed but increases torque.



Joint Axis Drive System

Why Can't We Connect the Motor Directly?

Without gears:

- Motor spins too fast.
- Robot arm moves uncontrollably.
- Torque is too low to lift objects.

With gears:

- Speed decreases.
- Torque increases.
- Robot moves slowly, smoothly, and can lift heavier loads.

Engr. Edwin C. Cuizon, Jr.

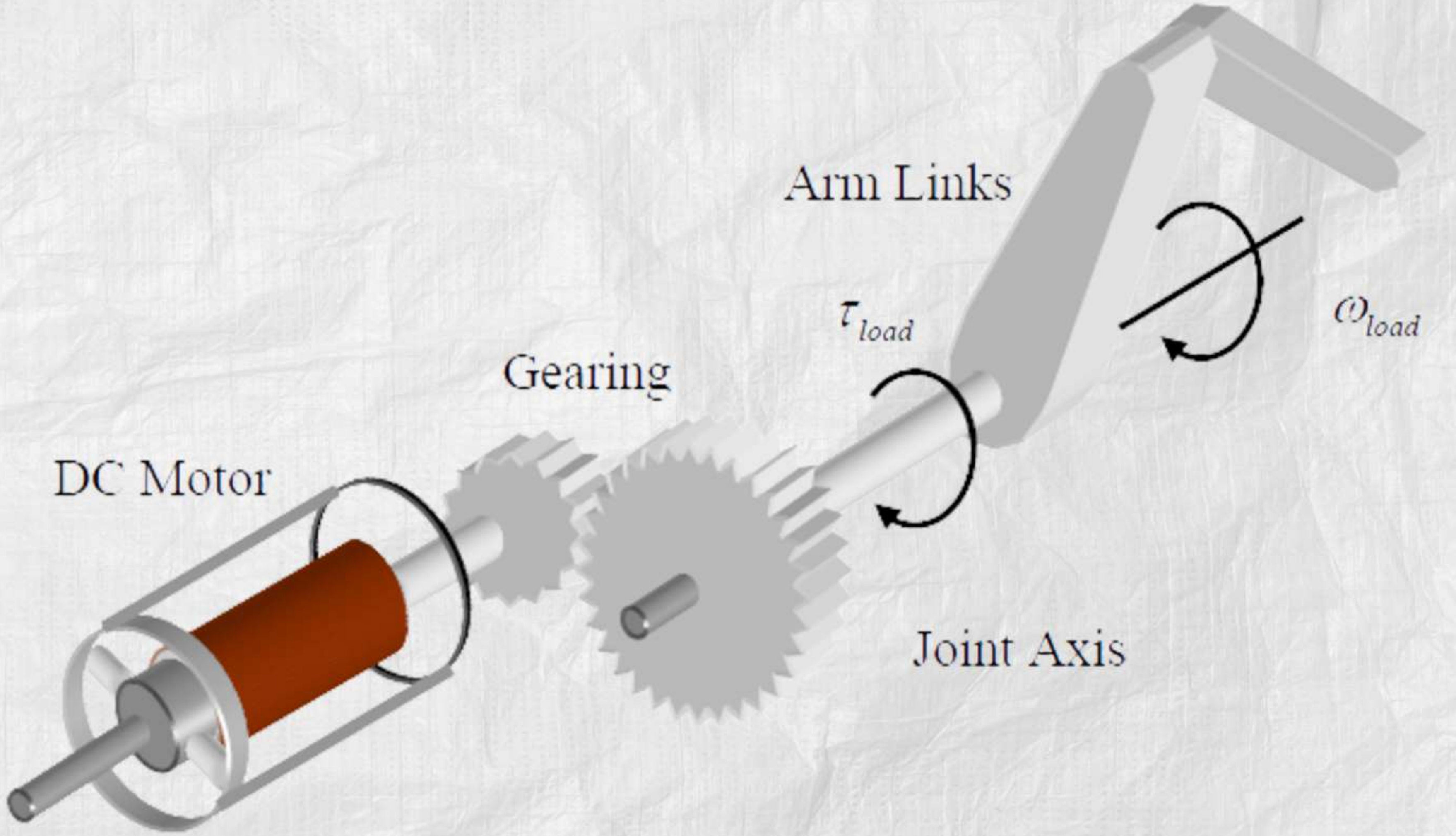
MOTOR IMPEDANCE

$$Z_m = \frac{torque}{angular\ velocity} = \frac{\tau_m}{\omega_m}$$

where:
- **Zm = motor impedance**
- **τm= motor torque**
- **ωm = motor speed**

***Impedance* means *resistance to movement or the balance* between strength and speed.**

***Mechanical impedance* describes how difficult it is to move a system.**



Joint Axis Drive System

GEAR RELATIONSHIP

$$\tau_{load} = r\tau_m \quad ; \quad \omega_{load} = \frac{\omega_m}{r}$$

where: r = gear reduction ratio

The gear does two things:

- **It multiplies torque.**
- **It reduces speed.**

IMPEDANCE AFTER GEAR

$$Z_{load} = r^2 Z_m$$

where: r = gear ratio

MOTOR ROTOR DYNAMICS

The motor rotor has inertia.

Newton's Second Law for Rotation states:

$$\tau = I\alpha$$

where:
- I = inertia
- α (alpha) = angular acceleration

MOTOR ROTOR DYNAMICS

$$I_m \dot{\omega}_m = \tau_m - \frac{\tau_{load}}{r}$$

where: r = gear ratio

Motor acceleration depends on:

- Motor torque pushing forward
- Load torque resisting motion

LOAD DYNAMICS

$$I_l \dot{\omega}_{load} = \tau_{load} - b\omega_{load}$$

where: $b\omega$ = **damping or friction**

I_l = **load inertia**

The arm accelerates because of torque, but friction tries to slow it down.

COMBINED DYNAMIC EQUATION

Combining motor dynamics and load dynamics gives:

$$I\dot{\omega}_{load} + B\omega_{load} = ku$$

where:

$I\dot{\omega}_{load}$ - effort needed to accelerate the arm

$B\omega_{load}$ - energy lost due to friction and damping

ku - power supplied by the motor

- The left side represents what resists motion.
- The right side represents the electrical input from the motor.

EFFECTIVE INERTIA

$$I = I_l + r^2 I_m$$

The total inertia is the inertia of the robot arm plus the motor inertia reflected through the gears.

EFFECTIVE DAMPING

$$B = b + r^2 \frac{K_m^2}{R}$$

This equation tells us that damping comes from two sources:

- Mechanical friction (b)
- The motor's electrical characteristics, amplified by the gear ratio.

A larger gear ratio increases damping, helping smooth the motion but also making the system respond more slowly.

INPUT GAIN

$$k = r\frac{K_t}{R}$$

A larger gear ratio increases the gain, meaning the motor can produce more useful torque at the robot joint.

$$I\dot{\omega}_{load} + B\omega_{load} = ku$$

EFFECTIVE INERTIA $$I = I_l + r^2 I_m$$

EFFECTIVE DAMPING $$B = b + r^2 \frac{K_m^2}{R}$$

INPUT GAIN $$k = r\frac{K_t}{R}$$

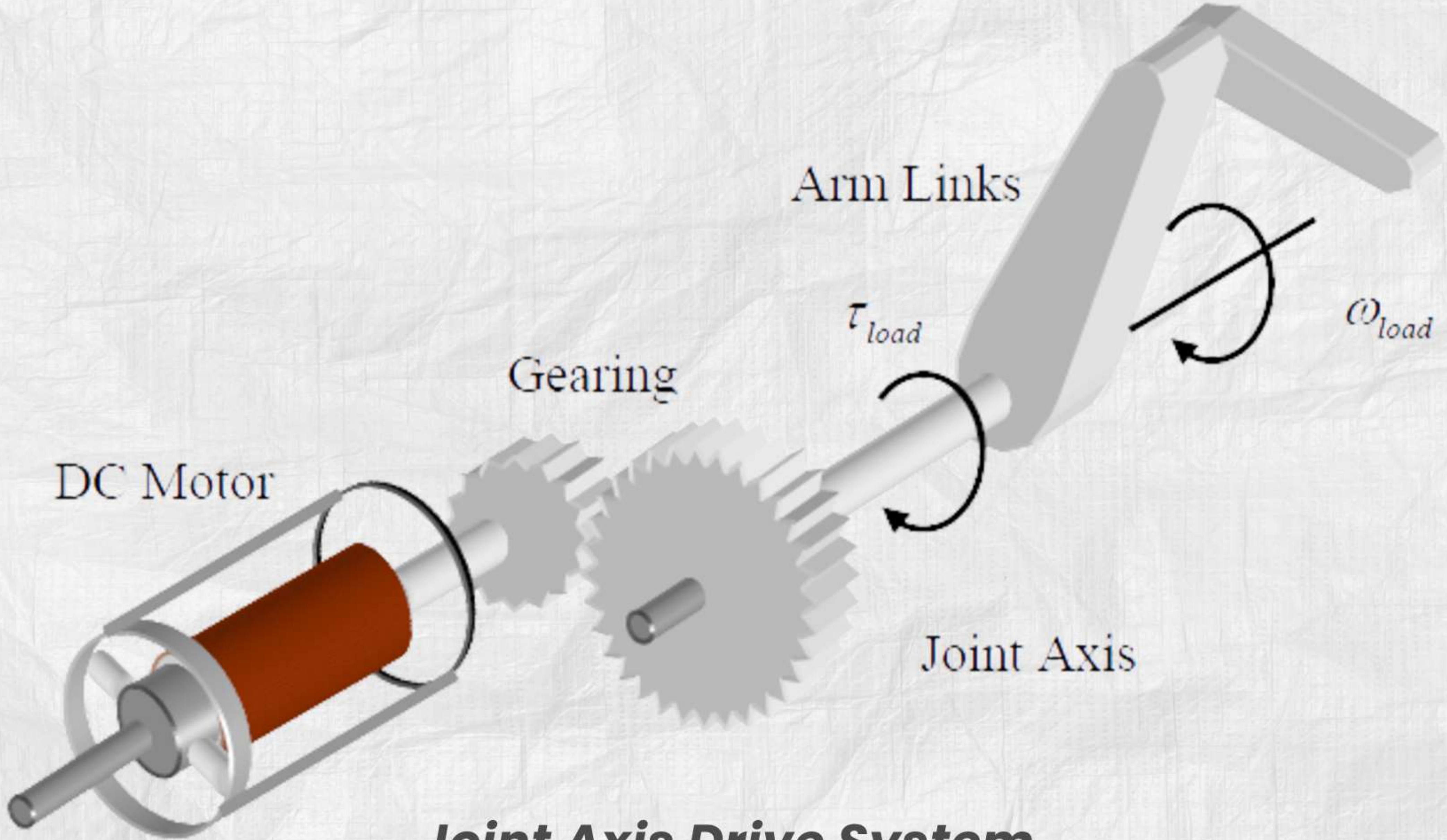
DYNAMICS OF SINGLE-AXIS DRIVE SYSTEMS

$$I\dot{\omega}_{load} + B\omega_{load} = ku$$

$I\omega_{load} + B\omega_{load} = ku$

- I – Effective inertia of the system (motor + load reflected to load side)
- B – Effective damping (friction + electrical damping effects)
- ω_{load} – Angular velocity of the load (rad/s)
- k – Input gain (motor torque gain factor)
- u – Input voltage or control input to the motor

DYNAMICS OF SINGLE-AXIS DRIVE SYSTEMS



Joint Axis Drive System

Electrical Energy

▼

DC Motor
(Fast speed, low torque)

▼

Gearing
(Reduces speed, increases torque)

▼

Joint Axis
(Rotates robot joint)

▼

Arm Link
(Moves the robot)

DISCUSSION QUESTION

A robotic joint uses a motor with inertia

$$I_m = 0.002\ kg \cdot m^2$$

and a gearbox ratio of r=40.

What is the reflected motor inertia at the load side?

DYNAMICS OF SINGLE-AXIS DRIVE SYSTEMS

Robots usually cannot connect motors directly to their joints because motors spin very fast but produce relatively small torque.

Key Points:

- ***Gears are used to increase torque and reduce speed.***
- ***A gear reducer helps the motor move heavy robotic arms.***
- ***Larger gear ratios provide more torque.***
- ***The motor and load together determine how quickly a robot joint can accelerate.***