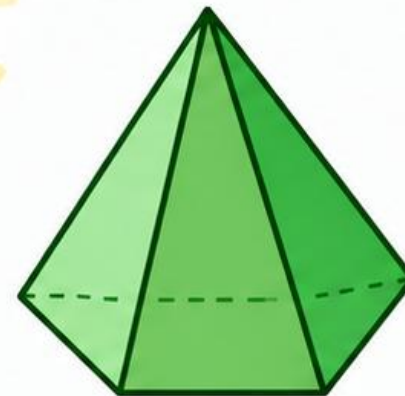


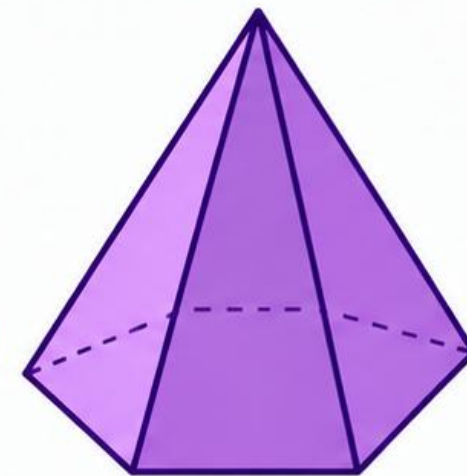
**Find the volume of pyramids  
other than square and  
rectangular pyramids.**



Triangular  
Pyramid



Pentagonal  
Pyramid



Hexagonal  
Pyramid

# The One Pyramid Volume Formula

(Works for ALL Bases)



## Vocabulary

**B**

### Base area (B):

The area of the shape on the bottom (triangle, pentagon, hexagon, etc.).

**h**

### Height (h):

The perpendicular distance from the base to the apex (tip).  
Not the slanted edge.

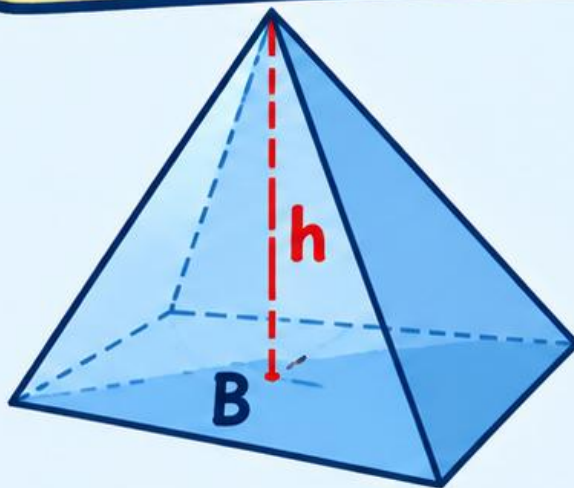
**V**

### Volume (V):

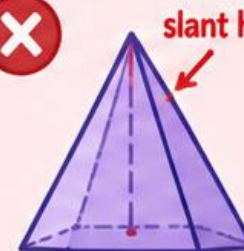
How much space is inside the pyramid (measured in cubic units).

## Universal Formula

$$V = \frac{1}{3} Bh$$



## Common Mistakes to Avoid



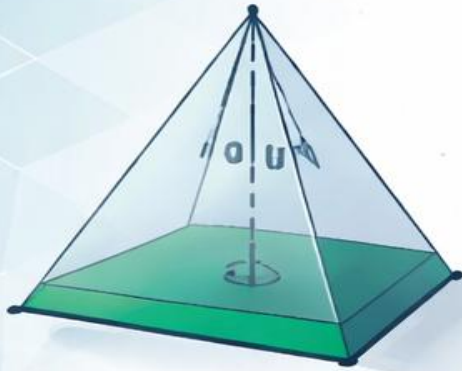
Using slant height instead of vertical height.



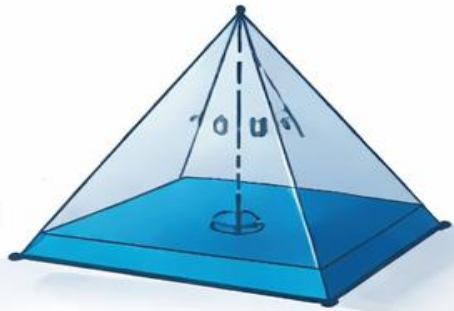
Forgetting to use square units for B and cubic units for V.

# The Big Idea: One Volume Formula Works for ALL Pyramids

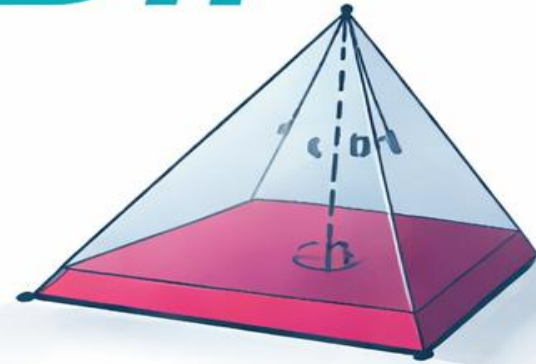
$$V = \frac{1}{3} Bh$$



**V** = volume  
(cubic units)



**B** = area of the base  
(square units)

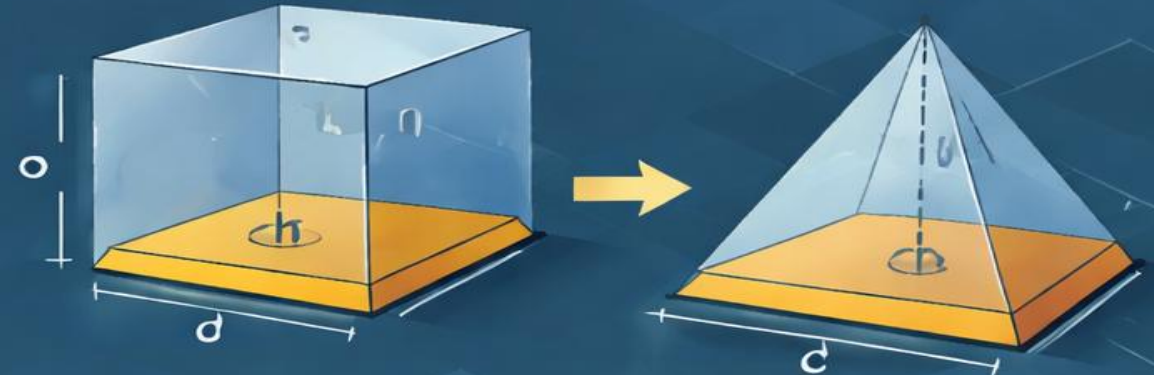


**B** = area of the base  
(square units)



**h** = height (units)  
(square units)

Why  $\frac{1}{3}$ ? A pyramid takes up one-third the volume of a prism with the same base area and height.



# HOW TO SOLVE ANY PYRAMID VOLUME PROBLEM

## (PROCESS)

1

Identify the base shape  
(triangle, pentagon, etc.)



Triangle



Square



Pentagon



Hexagon



Octagon

2

Find base area **B**  
using the correct area formula

B depends on the base shape

Triangle

$$B = \frac{1}{2}bh$$

Square

$$B = s^2$$

Pentagon

$$B = \frac{1}{2}ap$$

Hexagon

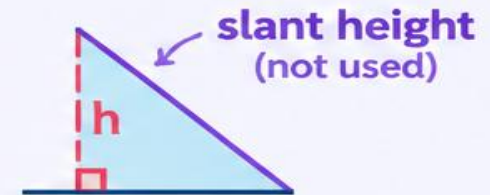
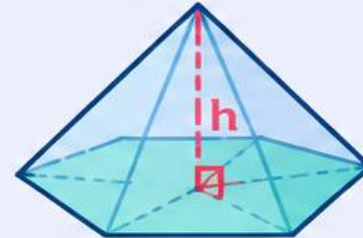
$$B = \frac{3\sqrt{3}}{2}s^2$$

(n-gon)

$$B = \frac{1}{2}ap$$

3

Use the perpendicular height **h**  
(not slant height)



4

Substitute into

$$V = \frac{1}{3} Bh$$

5

Label units as cubic

cm<sup>3</sup>

m<sup>3</sup>

ft<sup>3</sup>

in<sup>3</sup>

# Triangular Pyramid:

## Definition + Base Area

A triangular pyramid has a triangular base (often called a tetrahedron if all faces are triangles).

Base Area (triangle):

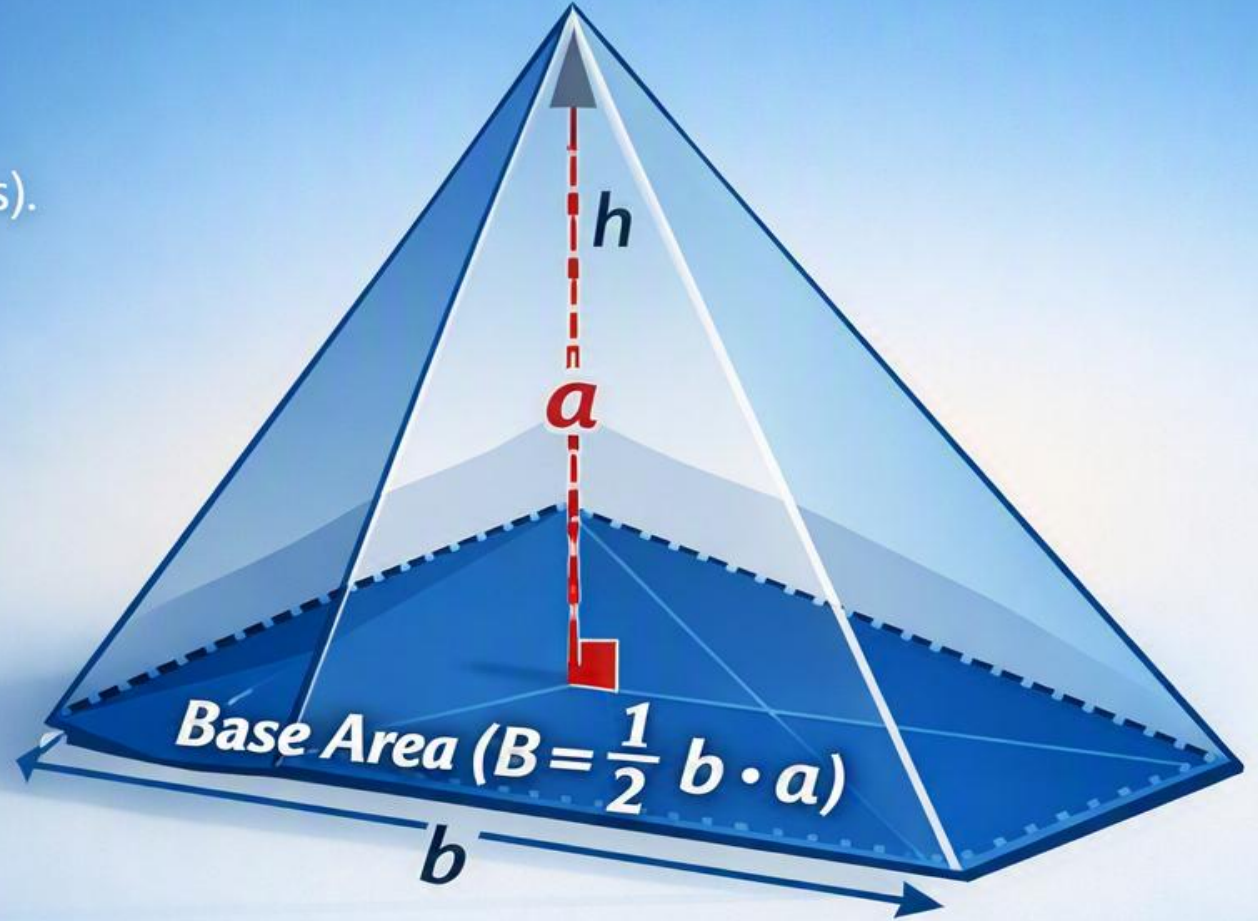
$$B = \frac{1}{2} b \cdot a$$

\*  $b$  = triangle base side length

\*  $a$  = triangle altitude (height of the triangle)

Pyramid Volume:

$$V = \frac{1}{3} B h$$



# Slide 6 Example: Triangular Pyramid Volume

- Triangular base has  $b = 10$  cm and triangle altitude  $a = 6$  cm
- Pyramid height  $h = 12$  cm

1. Base area:

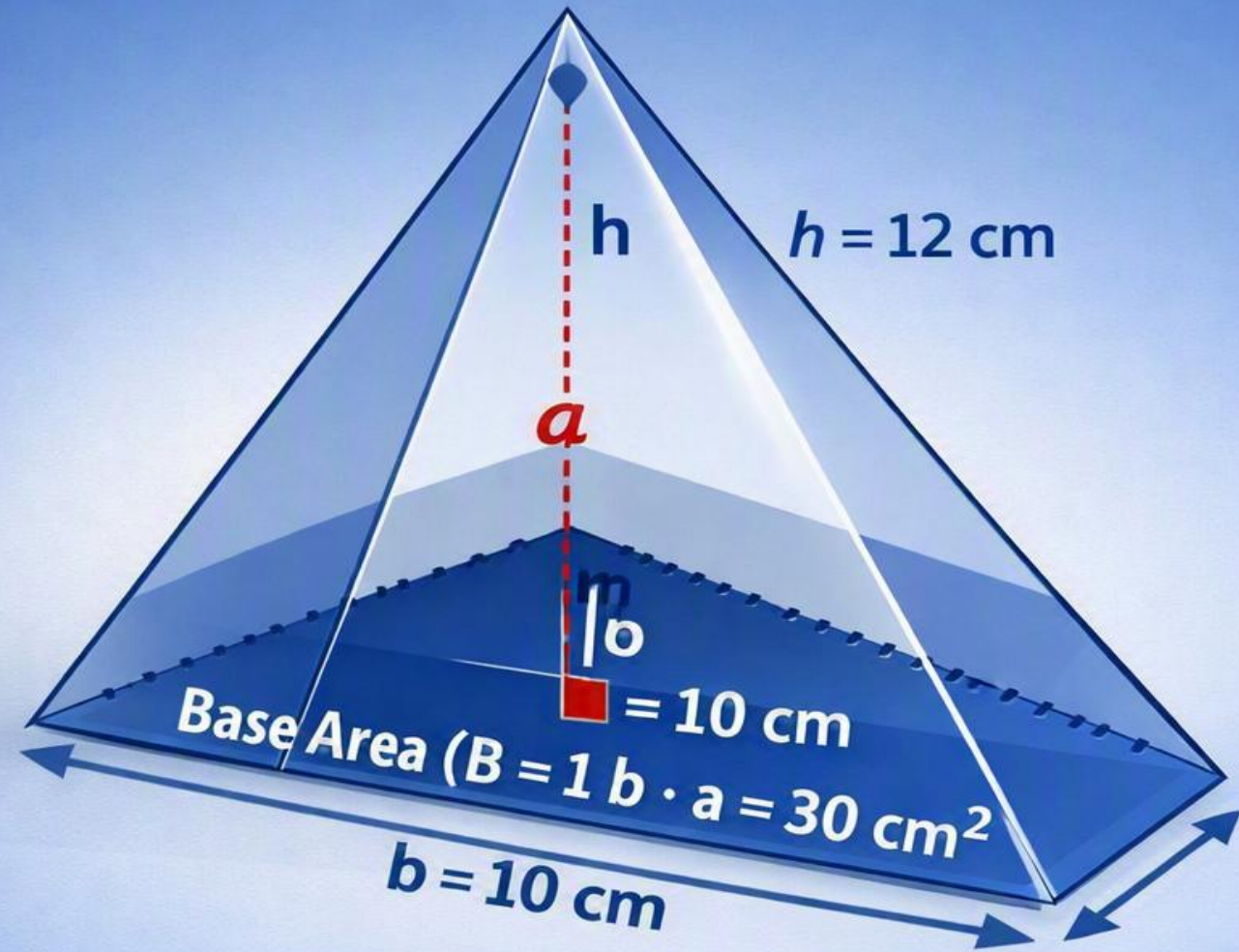
$$B = \frac{1}{2} (10)(6) = 30 \text{ cm}^2$$

\*  $b$  = triangle base side length

\*  $a$  = triangle altitude (*neight of the triangle*)

2. Volume:

$$V = \frac{1}{3} (30)(12) = \frac{360}{3} = 120 \text{ cm}^3$$



## Slide 7 Regular Polygon Pyramids (Pentagonal, Hexagonal, etc.)

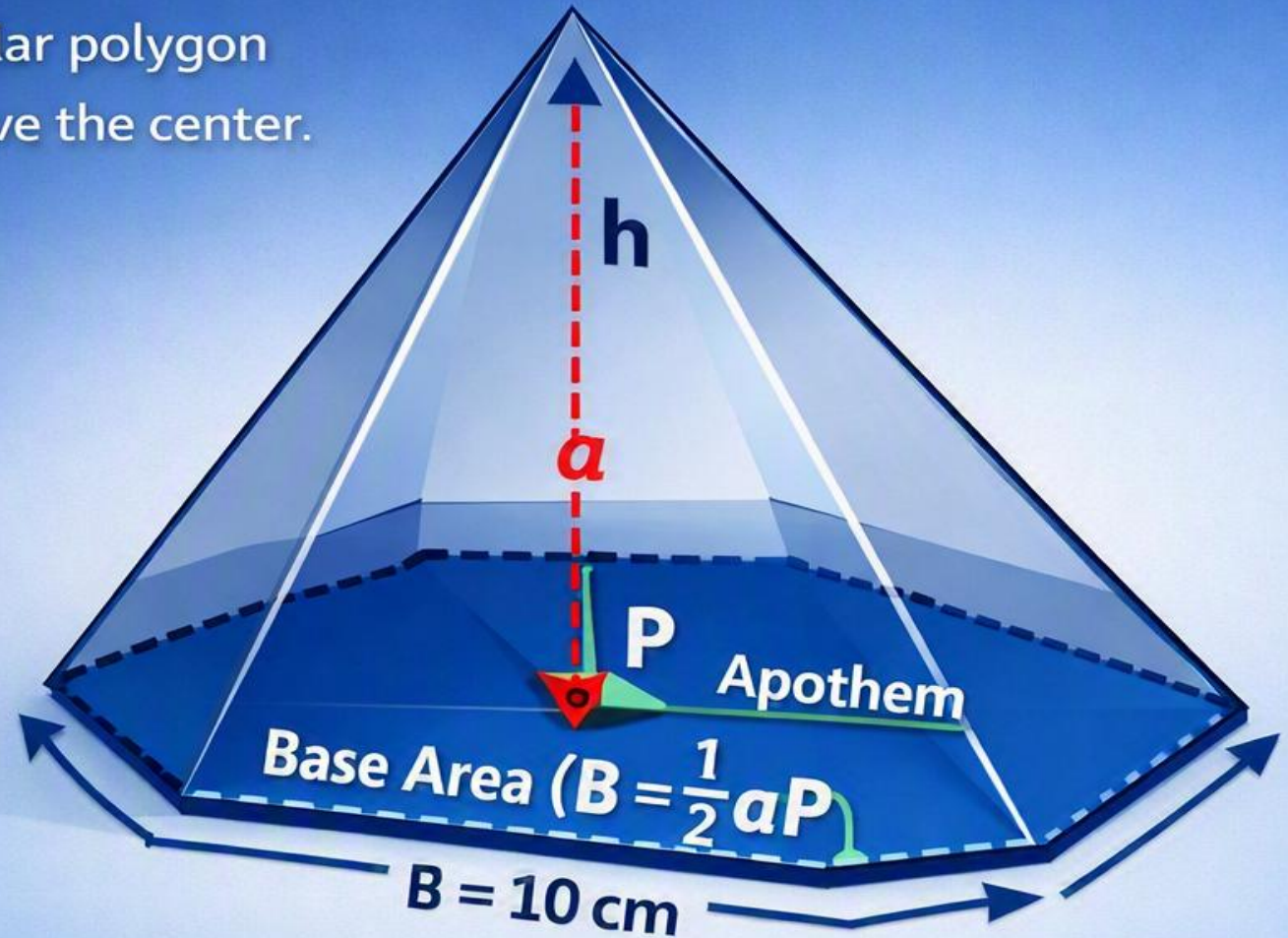
A regular pyramid has a base that is a regular polygon (all sides and angles equal) and an apex above the center.

**Perimeter  $P$ :**

$P$  = sum of side lengths

**Apothem  $a$ :**

$a$  = distance from center to the midpoint of a side



**Base Area:**

$$B = \frac{1}{2} aP$$

**Volume:**

$$V = \frac{1}{3} B h$$

# Comparing Pyramid Types:

## Triangular, Rectangular, and Regular Polygon



**Triangular Pyramid**

**Rectangular Pyramid**

**Regular Polygon Pyramid**

**Base:** Triangle

**Base:** Rectangle

**Base:** Regular Polygon

**Base Area:**  $B = \frac{1}{2} b \cdot a$

$B = l \cdot w$

$B = \frac{1}{3} a P$

**Volume:**  $V = \frac{1}{3} B h$

$V = \frac{1}{3} B h$

$V = \frac{1}{3} B h$

**Example:** *Tetrahedron*

*Square Pyramid*

*Pentagonal Pyramid*

## Example: Regular Hexagonal Pyramid Volume

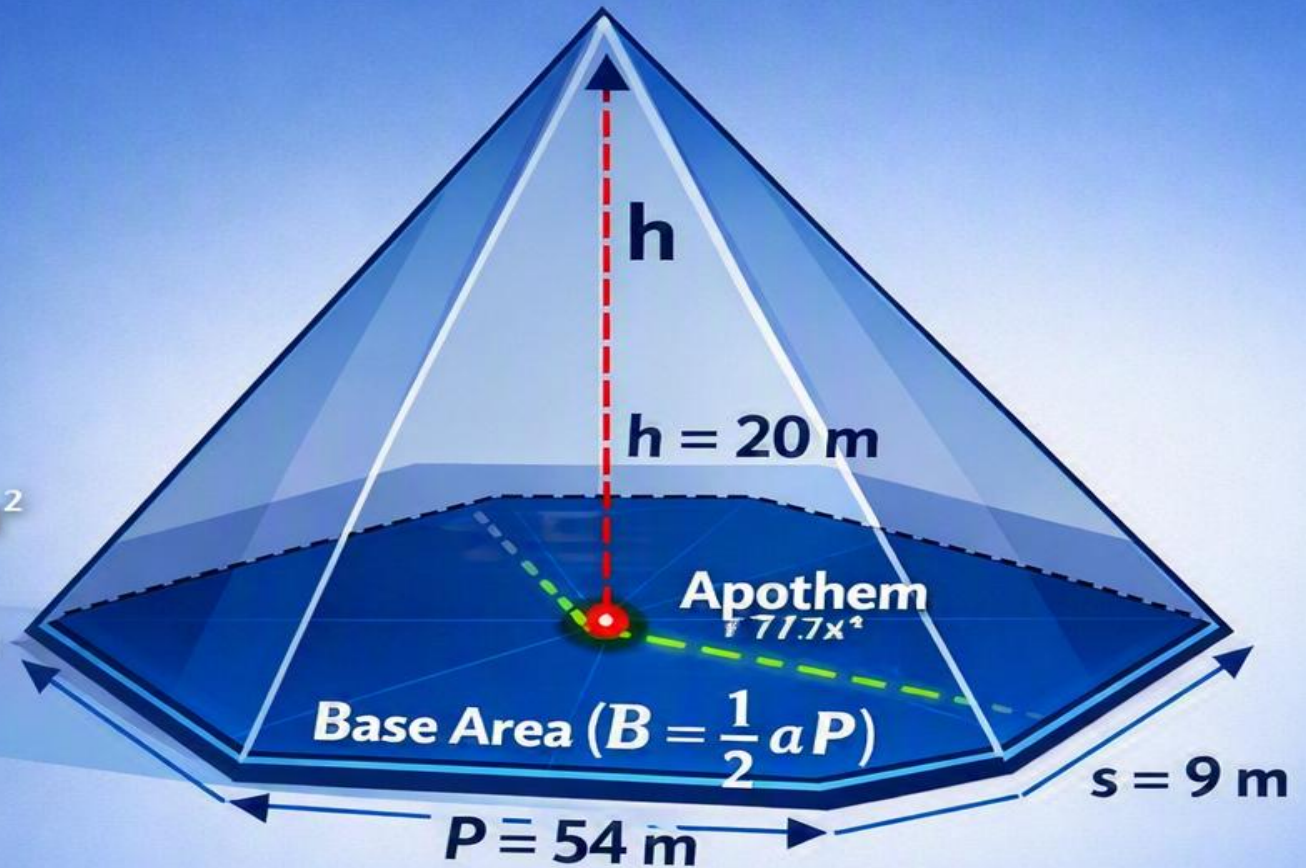
- Side length  $s = 9$  m
- Apothem  $a = 7.8$  m
- Pyramid height  $h = 20$  m

1. Perimeter:  $P = 6s = 6(9) = 54$  m

2. Base Area:  $B = \frac{1}{2}(7.8)(54) = 210.6$  m<sup>2</sup>

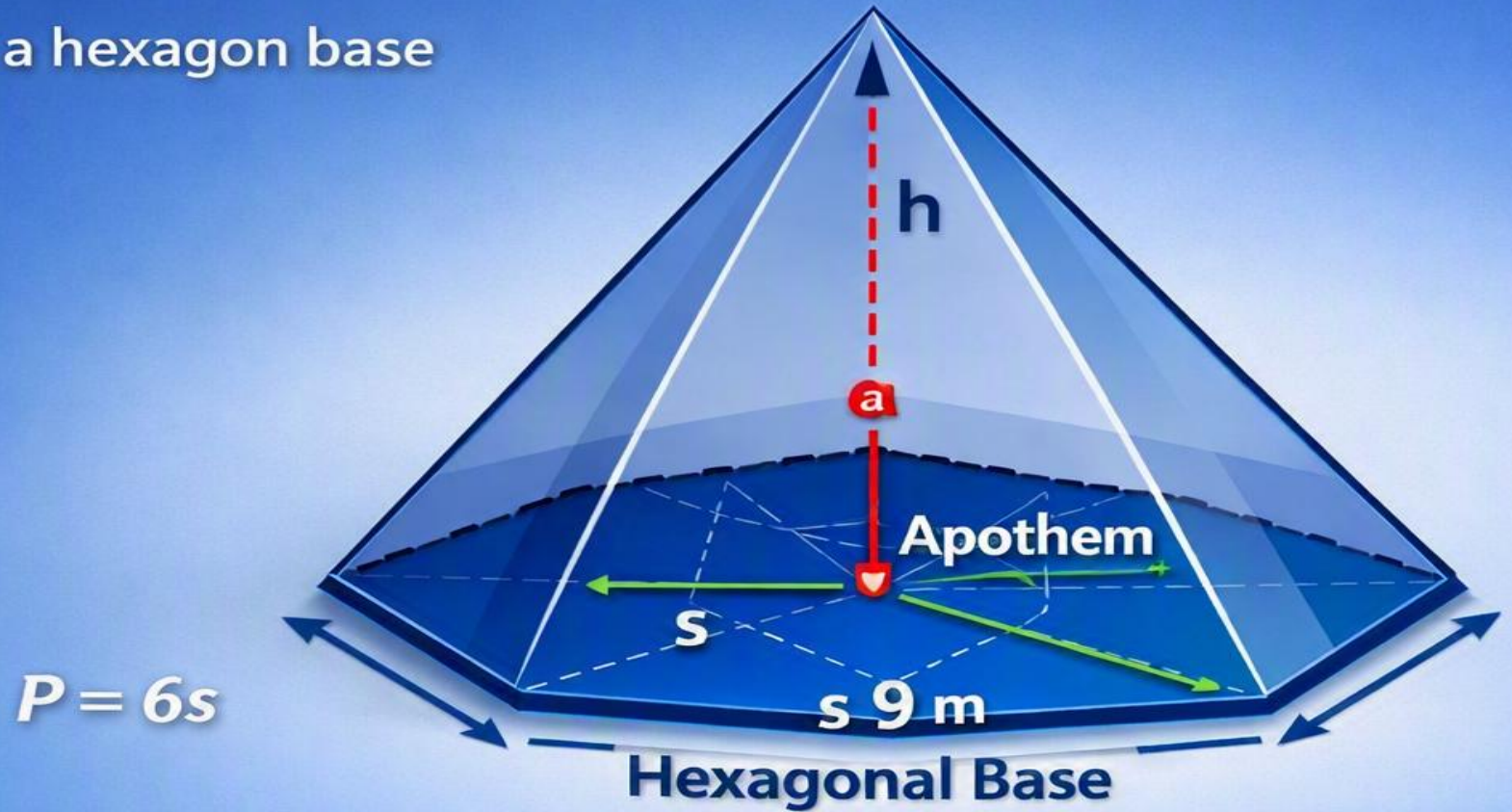
3. Volume:  $V = \frac{1}{3}(210.6)(20) = 1404$  m<sup>3</sup>

**Answer:  $V = 1404$  m<sup>3</sup>**



# Hexagonal Pyramid: Definition + Base Area

- A hexagonal pyramid has a hexagon base (often regular).
- **Perimeter:**  $P = 6s$
- **Base Area:**  $B = \frac{1}{2}aP$
- **Volume:**  $V = \frac{1}{3}Bh$



$$\text{Base Area: } B = \frac{1}{2}aP$$

$$\text{Volume: } V = \frac{1}{3}Bh$$

# Example: Regular Hexagonal Pyramid Volume

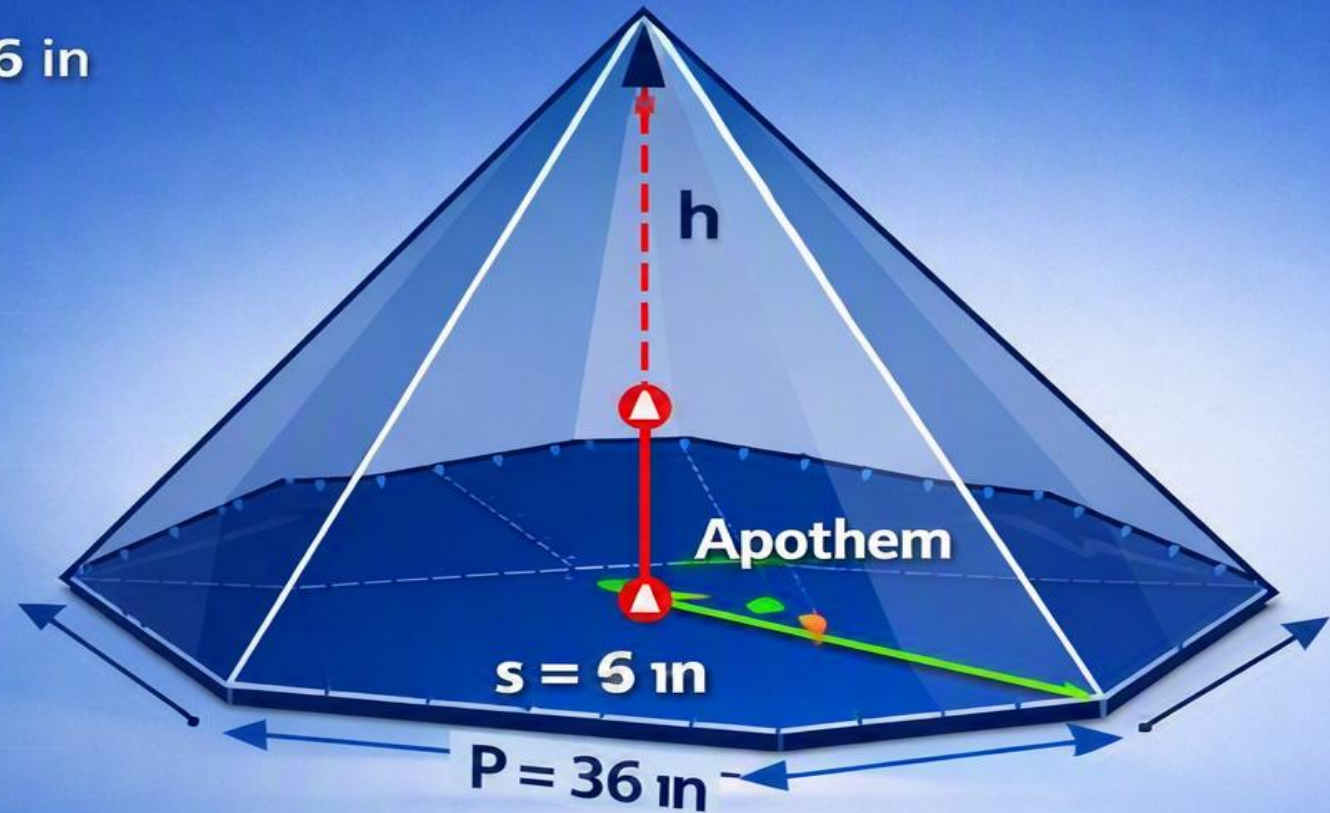
- Regular hexagon base: side length  $s = 6$  in
- Apothem  $a = 5.2$  in
- Pyramid height  $h = 10$  in

1. **Perimeter:**  $P = 6s = 36$  in

2. **Base Area:**  $B = \frac{1}{2}(5.2)(36) = 93.6$  in<sup>2</sup>

3. **Volume:**  $V = \frac{1}{3}(93.6)(10) = 312$  in<sup>3</sup>

**Answer: 312 in<sup>3</sup>**



$P = 36$  in

Base Area:  $B = 93.6$  in<sup>2</sup>

Volume:  $V = 312$  in<sup>3</sup>

# Trapezoidal Pyramid: Definition and Base Area

- A trapezoidal pyramid has a trapezoid base.

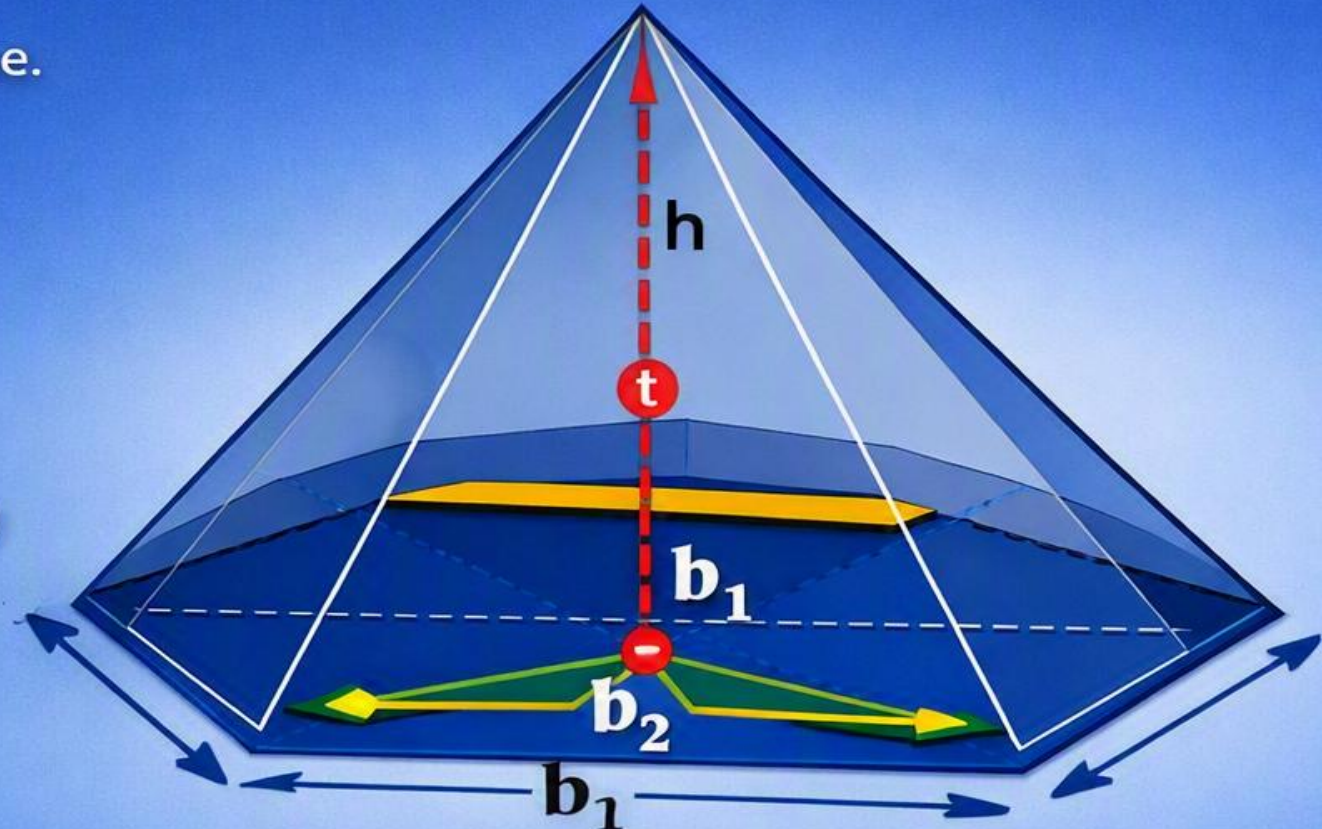
## Trapezoid Base Area:

$$B = \frac{1}{2}(b_1 + b_2)t$$

- \*  $b_1$   $b_2$  = lengths of the parallel sides
- \*  $t$  = trapezoid height (distance between parallel sides)

- **Pyramid Volume:**

$$V = \frac{1}{3} B h$$



Base Area:  $B = \frac{1}{2}(b_1 + b_2)t$

Volume:  $V = \frac{1}{3} B h$

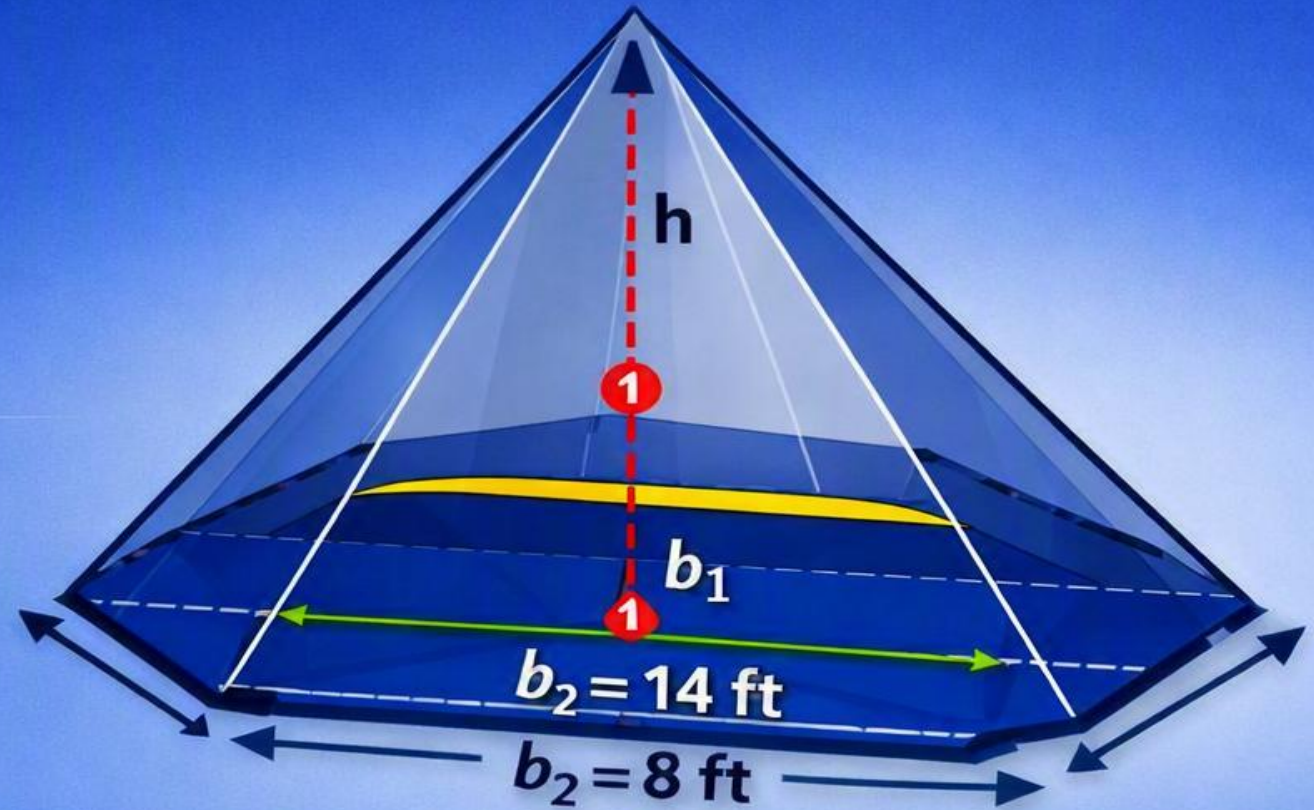
# Example: Trapezoidal Pyramid Volume

- Trapezoid base:  $b_1 = 14$  ft,  $b_2 = 8$  ft
- Trapezoid height  $t = 5$  ft
- Pyramid height  $h = 9$  ft

1. Base Area:  $B = \frac{1}{2} (14 + 8)(5)$   
 $= \frac{1}{2} (22)(5) = 55 \text{ ft}^2$

2. Volume:  $V = \frac{1}{3} (55)(9) = 165 \text{ ft}^3$

**Answer:  $165 \text{ ft}^3$**



Base Area:  $B = \frac{1}{3} (b_1 + b_2) t$

Volume:  $V = \frac{1}{3} B h$